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# Emergent gravity in the cubic tight-binding model of Weyl semimetal in the presence of elastic deformations

Alberto Cortijo [a], M.A. Zubkov [b,c,d,e,*]

[a] Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, 28049 Madrid, Spain
[b] ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia
[c] Moscow Institute of Physics and Technology, 9, Institutskii per., Dolgoprudny, Moscow Region, 141700, Russia
[d] Far Eastern Federal University, School of Biomedicine, 690950 Vladivostok, Russia
[e] National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, 115409 Moscow, Russia

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ABSTRACT

We consider the tight-binding model with cubic symmetry that may be relevant for the description of a certain class of Weyl semimetals. We take into account elastic deformations of the semimetal through the modification of hopping parameters. This modification results in the appearance of emergent gauge field and the coordinate dependent anisotropic Fermi velocity. The latter may be interpreted as emergent gravitational field.



## 1. Introduction

Recent experimental discovery of Dirac [1–6] and Weyl [7] semimetals enhanced essentially the development of the interdisciplinary field of research related to the interaction between the condensed matter physics and the high energy physics. Together with $^3$He-A [8] these materials are able to serve as an arena for the experimental investigation of various effects specific for the high energy physics [9–21] because the low energy effective theory that describes fermionic quasiparticles

* Corresponding author at: ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia.
*E-mail address:* zubkov@itep.ru (M.A. Zubkov).

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in Dirac and Weyl semimetals has an emergent relativistic invariance [8,22–25]. However, unlike the case of the superfluid Helium, the fermions in Dirac and Weyl semimetals are charged, and experience the external magnetic and electric fields. This facilitates, in particular, the investigation of effects related to chiral anomaly [3,26–30].

Within the high energy physics there exist certain difficulties related to the inclusion into consideration of the gravitational background. In particular, there is an ambiguity in the expressions for chiral anomaly in the presence of gravity with torsion. Different methods of calculation give different expressions (see Refs. [22,31–36], where those expressions are presented). At the same time, certain observable effects (mainly, in the condensed matter systems with emergent gravity) may be related to chiral anomaly in the presence of nontrivial geometry — for example, the appearance of Kopnin force acting on vortices in superfluid helium [8], the appearance of the effects of anomaly in Weyl semimetals [10,11,14–16,18–22,37,38], (in particular, chiral magnetic effect [39]), etc. Therefore, the theoretical and experimental investigation of Dirac and Weyl semimetals is extremely promising since it should be able to resolve principal internal problems of the high energy physics.

In the present paper we proceed our previous theoretical investigation of Dirac and Weyl semimetals [23–25]. We extend the consideration of the 2D graphene [40–47] to the 3D Weyl semimetals. We consider the particular cubic tight-binding model [48], which is able to describe qualitatively a certain class of real materials. Although this model does not describe any particular semimetal, we do not exclude, that our results may also be relevant for the quantitative study of some real materials in a certain approximation. The regular crystal, which is described by the given model represents a Weyl semimetal with two Fermi points. Each Fermi point hosts the two-component Weyl fermion. The chiralities of these two Weyl fermions are different. Such materials have more reach phenomenology, than the Dirac semimetals, where each Fermi point hosts the pair of Weyl fermions of opposite chirality (i.e. the massless Dirac spinor).

We discuss the situation, when elastic deformations are present that result in the appearance of the emergent gauge field and emergent gravity. We calculate both emergent gauge field and emergent vierbein and express them through the tensor of elastic deformation.

The paper is organized as follows. In Section 2 we describe the general algorithm for the calculation of the emergent fields in the presence of elastic deformations that was applied earlier to graphene. In Section 3 we recall the description of the unperturbed tight-binding model and demonstrate, that it indeed describes Weyl semimetal. In Section 4 we consider the modification of hopping parameters of the tight-binding Hamiltonian resulted from the elastic deformations. In Section 5 we calculate emergent gauge field and emergent gravitational field in the presence of elastic deformations. In Section 6 we describe the resulting low energy relativistic theory with the gauge field and the gravitational field. In Section 7 we end with the conclusions.

## 2. How do the emergent gravity and emergent gauge field appear in Weyl semimetals

Similar to the previous consideration of graphene [40–47,49] we calculate the effective low energy action in the given tight-binding model of Weyl semimetal as follows:

1. In the regular tight-binding model the positions of the Fermi points $\mathbf{K}^{(0)}$ are calculated as the points in momentum space, where the one-particle Hamiltonian vanishes. The Hamiltonians are expanded around each Fermi point up to the terms linear in $\mathbf{p} - \mathbf{K}^{(0)}$.
2. In the presence of elastic deformations the tight-binding model is modified. The terms in the Hamiltonian corresponding to the jumps between the adjacent lattice sites contain modified hopping parameters [23,48]. If those parameters would depend on the distance between the corresponding sites of the lattice only, then the relation between them and the tensor of elastic deformations would be similar to that of graphene [40–47]. Unlike graphene, however, the modification of hopping parameters in the 3D semimetals is more complicated (see [23,48]). We explore the modification of the hopping parameters following [23].
3. It is assumed, that the variations of hopping parameters are weak. Therefore, we may consider space as a composition of regions such that within each region the hopping parameters do not depend on coordinates. However, within each of such regions the hopping parameters are different for different directions of links.