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Tomograms for open quantum systems: In(finite) dimensional optical and spin systems

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HIGHLIGHTS

- Tomograms are constructed for open quantum systems.
- Finite and infinite dimensional quantum systems are studied.
- Finite dimensional systems (phase states, single & two qubit spin states) are studied.
- A dissipative harmonic oscillator is considered as an infinite dimensional system.
- Both pure dephasing as well as dissipation effects are studied.

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ABSTRACT

Tomograms are obtained as probability distributions and are used to reconstruct a quantum state from experimentally measured values. We study the evolution of tomograms for different quantum systems, both finite and infinite dimensional. In realistic experimental conditions, quantum states are exposed to the ambient environment and hence subject to effects like decoherence and dissipation, which are dealt with here, consistently, using the formalism of open quantum systems. This is extremely relevant from the perspective of experimental implementation and issues related to state reconstruction in quantum computation and communication. These considerations are also expected to affect the quasiprobability distribution obtained from experimentally generated tomograms and nonclassicality observed from them.

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1. Introduction

A quantum state can be characterized by a number of probability and quasiprobability distribution functions [1]. The quasiprobability distributions are not true probability distributions as most of them can have nonpositive values. Interestingly, this nonpositivity can be viewed as a signature of nonclassicality or quantumness. Specifically, the negative values of the Wigner [2] and P [3,4] functions serve as witnesses of nonclassicality. Further, zeros of Q function [5] also serve as witness of nonclassicality. As there does not exist any straight forward prescription for direct measurement of these quasiprobability distributions, several efforts have been made to construct measurable probability distributions that can be used to uniquely construct either all or some of these quasiprobability distributions. Such measurable probability distributions are referred to as tomograms [6–9]. In other words, the tomogram is a scheme for measuring a quantum state by using a representation in one to one correspondence with the true probability distribution rather than with a quasidistribution [10]. A relationship between a tomogram and a quasidistribution function, such as the Wigner function, can be established for both continuous and discrete systems [11,12]. Specifically, in Ref. [11] it was shown that quasiprobability distributions (P, Q, and Wigner functions) can be uniquely determined in terms of probability distributions for the rotated quadrature phase which can be viewed as an optical tomogram of the state. Similarly, in Ref. [12] it was shown that for finite dimensional phase states, discrete Wigner functions and tomograms are connected by a discretization of the continuous variable Radon transformation and was referred to as the *Plato transformation*.

In the recent past, a few successful attempts have been made to measure Wigner function directly in experiments [13,14], but the methods adopted are state specific. The same limitation is also valid for the theoretical proposals [15] for the measurement of Wigner function. Further, optical homodyne tomography has been employed for the experimental measurement of the Wigner functions of vacuum and squeezed states in [16,17], while distributions corresponding to Pegg–Barnett and Susskind–Glogower phase operators were also obtained in [17]. An experimental measurement of the P, Q and Wigner quantum phase distributions for the squeezed vacuum state has been reported in [18]. Precision of homodyne tomography technique was compared with conventional detection techniques in [19]. A number of alternative methods of tomography have also been proposed [20–22], and exploited to obtain phase distributions like Wigner and Q functions [23]. In [24] continuous variable quantum state tomography was reviewed from the perspective of quantum information. In brief, there does not exist any general prescription for direct experimental measurement of the Wigner function and other quasidistribution functions. In practice, to detect the nonclassicality in a system the Wigner function is obtained either by photon counting or from experimentally measured tomograms [15]. Thus, tomograms are very important for the identification of nonclassical character of a physical system. In another line of studies, simulation of quantum systems have been performed using tomography. For example, tomograms were used for simulation of tunneling [25–27] and multimode quantum states [28]. Attempts have also been made to understand the tomogram via path integrals [29,30].

Furthermore, how to reconstruct a quantum state from experimentally measured values is of prime interest for both quantum computation [13] and communication [31]. Specifically, in Ref. [13] it is strongly established that tomography and spectroscopy can be interpreted as dual forms of quantum computation, and in Ref. [31], quantum teleportation was experimentally performed over a distance of 143 km and the quality of teleportation was verified with the help of quantum process tomography (QPT) of quantum teleportation without feed-forward. Here it would be apt to note that QPT is an aspect of quantum state tomography in which a quantum process is obtained as a trace preserving positive linear map [32]. In the recent past, quantum process tomography has been discussed from the perspective of open quantum system effects [33–35]. A novel method of complete experimental characterization of quantum optical processes was introduced in [36]. It was further developed in [37,38] and extended to characterization of N-modes in [39]. In [40], QPT was applied to the characterization of optical memory based on electromagnetically induced transparency while [41] and [42] were devoted to QPT of the electromagnetic field and conditional state engineering, respectively. Quantum state tomography has its applications in quantum cryptography as well [43]. Specifically, in Ref. [43] an interesting protocol of quantum cryptography was proposed in which