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An exact thermodynamical model of power-law temperature time scaling

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ABSTRACT

In this paper a physical model for the anomalous temperature time evolution (decay) observed in complex thermodynamical system in presence of uniform heat source is provided. Measures involving temperatures T with power-law variation in time as $T(t) \propto t^{\beta}$ with $\beta \in \mathbb{R}$ shows a different evolution of the temperature time rate $\dot{T}(t)$ with respect to the temperature time-dependence $T(t)$. Indeed the temperature evolution is a power-law increasing function whereas the temperature time rate is a power-law decreasing function of time.

Such a behavior may be captured by a physical model that allows for a fast thermal energy diffusion close to the insulated location but must offer more resistance to the thermal energy flux as soon as the distance increases. In this paper this idea has been exploited showing that such thermodynamical system is represented by an heterogeneous one-dimensional distributed mass one with power-law spatial scaling of its physical properties. The model yields, exactly a power-law evolution (decay) of the temperature field in terms of a real exponent as $T \propto t^{\beta}$ (or $T \propto t^{-\beta}$) that is related to the power-law spatial scaling of the thermodynamical property of the system. The obtained relation yields a physical ground to the formulation of fractional-order generalization of the Fourier diffusion equation.

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1. Introduction

Thermal energy transfer due to phonon–phonon diffusion is very often observed in engineering and physical sciences leading to prediction of heat fluxes and temperature fields by means of the well-known Fourier transport equation. In this latter relation the instantaneous value of the thermal energy flux at any location is related to the local spatial gradient of the temperature [1]. The evolution of the temperature field provided by the Fourier transport equation is an exponential growth or decay and it proves to be accurate in several applications. However as soon as thermal energy flux is investigated in complex, multiphase and multiscale conductors and/or in presence of high frequency phenomena, then marked deviations of the exponential-type temperature evolution from experimental data may be observed [2,3]. Under these circumstances, several generalization of the Fourier equation have been proposed in the scientific literature from the mid of the last century [4]. Inertial correction to pure diffusive heat transport, including ballistic phononic transport, shows interesting features of the temperature field as the propagation of second-sound thermal waves observed in superfluids [5]–[6] and the pathological non-monotonic behavior of the entropy state function [7–9].

Fourier/Cattaneo models of thermal energy transport are not suitable, however, to describe the power-law rising of the temperature in recent challenging applications. Indeed an anomalous evolution of the temperature field has been observed in ultrafast phenomena as the laser pulsatile radiation in biological tissues [10] as well as in thermal energy transport in nanostructured materials [11].

In this context generalization of the Fourier transport equation has been proposed replacing classical differential operators with their real-order (fractional) counterparts $\frac{d^j f}{dt^j} = \left(D^j f\right)(t) \rightarrow \frac{d^\beta f}{dt^\beta} = (D^\beta f)(t)$ with $\beta \in \mathbb{R}$ [12].

This approach has been used in several context of physics and engineering yielding the so-called fractional-order Fourier transport equation [13–16] or the non-local fractional-order thermodynamics [17–21].

Despite the wide success beyond the introduction of fractional-order Fourier equations it has been only presented on phenomenological basis and no thermodynamical systems have been provided, to the best of the author's knowledge, with power-law time scaling of the temperature.

This paper aims to fill this gap introducing an heterogeneous conductor that yields, exactly, the anomalous time evolution of the temperature field as $T(t) \propto t^\beta$ and $\beta \in [0, 1]$. It is shown that, as the thermodynamical properties of the system vary as power-law of the distance from the insulated border, a relation among the thermal properties of the conductor and the exponent of the decay of the temperature field is obtained.

A similar feature was first encountered in the field of classical mechanics where the anomalous material creep/relaxation has been modeled with a proper mechanical ladder yielding the exact description of material hereditariness [22–27]. In other studies the presence of anomalous evolution of pressure and/or mass flux has been related to the transport across fractal porous materials [28–30]. In other studies the physical representation of the spatial interactions involved in the use of fractional-order calculus has been investigated (see e.g. [31–33]) also in the context of long-range thermal energy fluxes [34–36]).

The paper is organized as follows: In the next section a discrete thermodynamical system will be shown to approximate the power-law evolution of the temperature field. In Section 3 the non-homogeneous continuous thermodynamical system representation is reported and it is shown to describe, exactly, the power-law evolution of the temperature with exponent related to the heterogeneity of the material decay. Some conclusions are reported in Section 4 whereas details on fractional-order operators generaling classical operators have been reported in the Appendix.

2. A discrete mass representation of the anomalous temperature evolution

The idea described in this paper stems out from a physical consideration about the temperature rising observed in complex thermodynamical systems as biological tissues, nanostructured materials as well as in presence of multiphasic materials in which different kinds of thermal energy carriers