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Constraints on operator ordering from third quantization

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ABSTRACT

In this paper, we analyse the Wheeler–DeWitt equation in the third quantized formalism. We will demonstrate that for certain operator ordering, the early stages of the universe are dominated by quantum fluctuations, and the universe becomes classical at later stages during the cosmic expansion. This is physically expected, if the universe is formed from quantum fluctuations in the third quantized formalism. So, we will argue that this physical requirement can be used to constrain the form of the operator ordering chosen. We will explicitly demonstrate this to be the case for two different cosmological models.

1. Introduction

The wave function of the universe contains all the physical information about the universe because it describes the quantum state of the universe [1,2]. In the no-boundary proposal the wave function of the universe is obtained by summing over all four geometries and field configurations that match a specific field configuration on a spatial section. The wave function can also be obtained as a solution to the Wheeler–DeWitt equation, which can be viewed as the Schrödinger's equation for gravity [3,4]. However, just as the single particle Schrödinger's equation cannot be used to analyse a multi-particle system in the first quantized formalism, the Wheeler–DeWitt equation cannot be used to describe

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multi-universe system in the second quantized formalism. However, a multi-particle system can be analysed by using second quantization. This observation has motivated the study of third quantization of the Wheeler–DeWitt equation [5–10]. In the third quantized formalism, the Wheeler–DeWitt equation is viewed as a classical field equation. Thus, an action is constructed such that the field equations corresponding to that action are the Wheeler–DeWitt equation. This action describes the theory that can be third quantized. The third quantization of this action produces a multi-geometry theory. If these geometries are identified with individual universes, then the third quantization of this action describes a multiverse [11–16].

The third quantization has also been used for analysing the virtual black holes [17]. Here the fluctuations of the spacetime at Planck scale cause the formation of virtual black holes. This model of virtual black holes can be used to solve the problem of time. This is because in this model the entropy of the universe keeps increasing due to the interaction of these virtual black holes with matter. The direction of time can then be identified with the increase of the total entropy of the universe. The model for the spacetime foam can also be used to explain the end stage of the evaporation of real black holes [18]. In this model, real black holes evaporate down to Planck size and then disappear in the sea of virtual black holes. It may be noted that the third quantization has also been used to address the cosmological constant problem using the idea of baby universes [19]. In this model, the creation or annihilation of baby of universe in the third quantized formalism is similar to the creation or annihilation of a particle in the second quantized formalism. The propagator of the theory corresponded to a wormhole, and the third quantized version of the momentum conservation is represented by the conservation of the axion charge.

It is possible for the universe to form from quantum fluctuations in the third quantized formalism [20]. Thus, it is expected that the early stages in the evolution of the universe would be dominated by quantum fluctuations. It is known that the geometry of spacetime is described by a classical spacetime at later stages, so it is expected that the quantum fluctuations will get minimized at later stages of the cosmic expansion. The uncertainty for a model of third quantized universes has been studied, and it was observed that the fluctuations in the third quantized formalism decrease very rapidly during the course of cosmic expansion [21,22]. This uncertainty has also been studied for third quantized Brans–Dicke theories [23]. In this analysis, the distribution function for the universes has been obtained. The uncertainty principle has also been discussed in the context of the third quantization of $f(R)$ gravity theories [24,25]. The distribution function for the universes in the third quantization Kaluza–Klein theories has also been obtained [26]. In this analysis, it was demonstrated that the compactification of geometries is consistent with third quantization. The third quantization has been used to analyse the quantum transitions from the string perturbative vacuum to cosmological configurations which is characterized by isotropic contraction and decreasing dilaton [27]. It was observed that such transitions could be represented by the production of pairs of universes from the vacuum state. All these analyses were done for specific choice of operator ordering. However, it is known that operator ordering can have direct physical consequences [28,29]. So, this motivates us to study the effect of operator ordering on the third quantization, and this is what will be done in this paper. We would like to point out that the main aim of this paper is to use the physical requirements on cosmological models to restrict the form of operator ordering used. This is because there is no mathematical way to prefer one choice of factor ordering from another.

In this paper, we study the effect of operator ordering in the third quantized formalism. We will use the third quantized formalism for analysing the effect of uncertainty relation on the structure of spacetime during cosmic expansion. We will also discuss the operator ordering for this theory. We will observe that for a specific choice of operator ordering quantum fluctuations dominate at the early stage of the universe and the spacetime becomes classical at the later stages. This is physically expected, if the universe is formed from quantum fluctuations in the third quantized formalism. The remaining part of the paper is organized as follows. In Section 2, we will analyse the third quantization of general relativity with a cosmological constant. In Section 3, we will then study the uncertainty relation for this model, and in Section 4, we will analyse the operator ordering for this model. Then in Section 5, we will apply this formalism for another minisuperspace model. Finally, we will summarize our results in Section 6. We will also suggest some possible extension of this works in this last section.