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Thermal atom–atom entanglement in a bichromatic Kerr nonlinear coupler

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HIGHLIGHTS

- Thermal atom–atom entanglement in a Kerr nonlinear coupler is investigated.
- The atom–atom, field–field and atom–field interactions are considered.
- The entanglement can be tuned by atom–field structure parameters.
- The entanglement starts from zero and terminates at a finite temperature.

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ABSTRACT

In this paper thermal entanglement between two identical two-level atoms within a bichromatic cavity including Kerr nonlinear coupler is investigated. In this study, besides atom–field interaction, the field–field (via linear and Kerr-type couplings) and atomic dipole–dipole interactions are also included. It is also assumed that the cavity is held at a temperature T, so that all atom–photon states with probabilities defined by Boltzmann factor are present. Using a canonical transformation, the presented model is converted to a generalized form of Jaynes–Cummings model. After introducing Casimir operators of the system, it is shown that the Hamiltonian representation is block-diagonal. Diagonalizing each block, the thermal (Gibb's) density matrix, written in the bases of total Hamiltonian, is obtained. The reduced atomic density matrix and consequently the concurrence, as a measure of entanglement, are obtained by partial tracing of thermal density matrix over the bichromatic photonic states. The concurrence vanishes at zero temperature, indicating that the ground state is separable, exhibits a maximal at a critical temperature and terminates at a finite temperature. The influences of coupler nonlinearities and

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dipole–dipole coupling on the thermal atom–atom entanglement are also addressed in detail.

1. Introduction

Entanglement, one of the most fundamental and intriguing features of composite quantum systems, has been known as the main resource of quantum information processing (QIP) [1–5]. Therefore, it is important to explore means for creating and manipulating entanglement. In recent years different ways have been proposed to realize entangled states by means of atom–photon interaction [6–9]. The simplest way to study the atom—field interaction is the Jaynes–Cummings model (JCM). The generalizations of the JCM for the two-atom cases have also attracted considerable interest [10–12] because the two-level atom can be considered as the qubit—the basic unit of quantum information. Furthermore, the entanglement between the atoms and the field has been studied by assuming that this interaction occurs in a cavity filled with a dielectric medium [13,14]. On the other hand, Kerr nonlinear couplers can be created using optical fibers or photonic crystals and the amount of photonic couplings can be adjusted by controlling the interaction length and medium characteristics [15–17]. However, if it is assumed that the atoms are located inside a Kerr nonlinear coupler, besides direct atomic dipole–dipole interaction [18], the photonic linear and Kerr-type couplings have also appeared. As a result, the atoms (photons) interact reciprocally as well as with both photons (atoms). These interactions strongly impress the atom–atom, atom–photon and the photon–photon entanglement.

The atom–atom entanglement via coherent field at absolute zero temperature and via thermal field at finite temperature has been the subject of several reports [18–20]. However, due to the interaction with the environment, as a heat reservoir, the quantum systems, as a whole, including both atoms *and* photons, are always in mixed states [13,21]. Therefore, temperature is of crucial importance in the field of QIP and leads disentanglement [22] or vice versa [23]. Assuming thermal equilibrium with the environment, transition tendency from an atom–photon state to another one should be counterbalanced by transition tendency in the reverse direction. This equality of tendencies is usually referred to *detailed balance* once thermal equilibrium is established [24,25]. In thermal equilibrium with the environment, all of the atom–photon states with probabilities, specified by Boltzmann factor, are present. Furthermore, in this paper the lossless Kerr nonlinear coupler has been assumed, thus the time evolution of density operator obeys von Neumann equation of motion [26]. Anyway, in this paper, a model that describes two identical two-level atoms as two qubits interacting with a bichromatic field inside a lossless Kerr nonlinear coupler in the presence of the atomic dipole–dipole coupling at an equilibrium temperature T is investigated. Apart from other new features of this work, in particular, the effect of Kerr-type coupling and atomic dipole–dipole coupling on the atom–atom entanglement will be addressed in detail. In general, this system gives more generalize and realistic features of atom–atom entanglement and introduces a better understanding of entanglement.

In order to study the thermal entanglement, the concurrence, based on the entanglement of formation [27], is a useful quantity that leads to the amount of entanglement [28] especially for the two two-level atom cases. To pursue this aim, the model along with the corresponding Hamiltonian is introduced. It is then shown that the matrix representation of the Hamiltonian is block-diagonal with ever-growing dimensions whose dimensionalities depend upon the eigenvalue of the total atom–photon excitation operator, $\mathbb{N}$, which is denoted by N. Applying a canonical photonic transformation, each block is reduced to a 1×1, a 3×3 and $N - 1$ 4×4 subblocks. Therefore, this transformation enables us to diagonalize the transformed Hamiltonian and calculate the eigenvalues in general. Using the eigenvalues and the eigenvectors of the Hamiltonian, the thermal (Gibb's) density operator (matrix), as a function of temperature, is then calculated. Tracing over the bichromatic photonic states, thermal atomic density matrix and then the concurrence are obtained and consequently the roles of the atomic dipole–dipole coupling and the photonic nonlinearities are recognized.