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Self-force on an electric dipole in the spacetime of a cosmic string

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HIGHLIGHTS

- Review of regularized Green's function applied to the problem.
- Self-force on an electric dipole in the string spacetime for some orientations.
- Representation via graphs of the self-forces versus angular parameter of the cosmic string.
- Self-force induced by the string seen as an interaction between two dipoles.
- Discussion about the superposition principle in this non-trivial background.

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ABSTRACT

We calculate the electrostatic self-force on an electric dipole in the spacetime generated by a static, thin, infinite and straight cosmic string. The electric dipole is held fixed in different configurations, namely, parallel, perpendicular to the cosmic string and oriented along the azimuthal direction around this topological defect, which is stretched along the z axis. We show that the self-force is equivalent to an interaction of the electric dipole with an effective dipole moment which depends on the linear mass density of the cosmic string and on the configuration. The plots of the self-forces as functions of the parameter which determines the angular deficit of the cosmic string are shown for those different configurations.

1. Introduction

The phenomenon of self-interaction force experienced by an electric charge placed in an arbitrary gravitational field was analyzed for the first time by DeWitt and Brehme [1]. In this scenario, the self-

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force is induced by the interaction of the electromagnetic field generated by the electric charge and the curvature of the spacetime [2–9]. This interaction modifies the electrostatic potential in such a way that the electric charge experiences a finite self-force. This is an example of how the self-force on a charged particle held fixed in a gravitational field depends on the local aspects (geometry) of the spacetime.

In fact, the self-interaction force depends not only on the geometry of the spacetime but also on the global features of the spacetime. This can be understood as a manifestation of non-local (topological) influence of gravity on the electromagnetic field, and means that non-local aspects of the gravitational fields are of fundamental importance in describing this physical system. In other words, the intrinsic geometry of the space is not sufficient to describe completely the interaction of the electromagnetic field produced by an electric charge and the gravitational field.

A scenario in which we can investigate the topological features of the interaction between an electromagnetic and a gravitational field is the spacetime produced by a thin, infinite and straight cosmic string [10–13], which looks like the direct product of the two-dimensional Minkowski space and a cone, and thus it is locally flat but not globally. The metric outside a cosmic string positioned along of the z axis can be written, in cylindrical coordinates, as

$$ds^2 = dt^2 - d\rho^2 - \frac{\rho^2}{\nu^2}d\phi^2 - dz^2, \tag{1}$$

where the domain of the azimuthal angle is $0 \leq \phi \leq 2\pi$. The parameter ν is given by $\nu = (1-4G\mu)^{-1}$, where μ is the mass per unit length of the string. Note that, we can redefine the azimuthal angle in such a way that $\theta = \frac{1}{\nu}\phi$, with $0 \leq \theta < 2\pi/\nu$. Thus, the metric expressed in Eq. (1) is actually a locally flat one, which means that the spacetime of an infinitely thin cosmic string may be regarded as the Minkowski spacetime with a deficit angle $2\pi(1-\nu^{-1})$.

This lack of global flatness of the cosmic string spacetime originates an electrostatic self-force on a charged particle, investigated in [14–17], on a linear charge distribution [18], on a mean charge density [19], on an electric dipole in the presence of a point mass and in the context of planar gravity [20], or on an electric dipole held fixed parallel to the cosmic string [21,22]. The self-forces on a particle [23] and on a distribution of charges [24], placed in other locally Minkowski space–times, also were analyzed.

The purpose of this work is to investigate how the self-force phenomenon induced by the global features of the gravitational field of a cosmic string manifests itself on an electric dipole held fixed in different configurations, as for example, parallel, perpendicular to the cosmic string and oriented along the azimuthal direction (orthogonal), and in what way these forces depend on the parameter which defines the deficit angle of the spacetime under consideration.

A similar problem has been already investigated using the retarded Green's function [25] associated with the electromagnetic field of a electric dipole and of other static distributions of charge, in the spacetime of a straight, thin and infinite cosmic string. It was shown that these sources are influenced by the cosmic string through a self-interaction with their fields. In our paper, the self-interaction on an electric dipole placed in the cosmic string spacetime was derived by a different method which was the same one used in [14].

This paper is organized as follows: in Section 2 we revise the renormalized Green's function method, necessary for the calculation of self-energy on an electric charge. In Sections 3–5 we obtain the self-energy and use this result to calculate the self-force on a point electric dipole when it is parallel, perpendicular and orthogonal to the cosmic string, respectively. We also obtain for each configuration the graphs which describe how the self-force depends on the parameter which defines the deficit angle. We discuss our results in Section 6.

2. The regularized Green's method revisited

The volumetric charge distribution of an electric dipole with point charges q and $-q$, separated by a distance $\mathbf{a}$, is given by

$$\rho(\mathbf{x}) = \rho_q(\mathbf{x}) + \rho_{-q}(\mathbf{x}) = q[\delta^3(\mathbf{x}-\mathbf{x}_1) - \delta^3(\mathbf{x}-\mathbf{x}_2)], \tag{2}$$