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Quantum mechanics in noninertial reference frames: Violations of the nonrelativistic equivalence principle

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HIGHLIGHTS

- A formulation of Galilean quantum mechanics in non-inertial reference frames is given.
- The key concept is *the Galilean line group*, an infinite dimensional group.
- A large class of general cocycle representations of the Galilean line group is constructed.
- These representations show violations of the equivalence principle at the quantum level.
- At the classical limit, no violations of the equivalence principle are detected.

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ABSTRACT

In previous work we have developed a formulation of quantum mechanics in non-inertial reference frames. This formulation is grounded in a class of unitary cocycle representations of what we have called the Galilean line group, the generalization of the Galilei group that includes transformations amongst non-inertial reference frames. These representations show that in quantum mechanics, just as is the case in classical mechanics, the transformations to accelerating reference frames give rise to fictitious forces. A special feature of these previously constructed representations is that they all respect the non-relativistic equivalence principle, wherein the fictitious forces associated with linear acceleration can equivalently be described by gravitational forces. In this paper we exhibit a large class of cocycle representations of the Galilean line group that violate the equivalence principle. Nevertheless the classical

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mechanics analogue of these cocycle representations all respect the equivalence principle.

1. Introduction

This paper is a sequel to two previous papers [1,2] on the formulation of Galilean quantum mechanics in non-inertial reference frames and its implications for the equivalence principle and gravity. Both of these papers were grounded on the same philosophical principle, namely that the structure of quantum physics is fundamentally determined by the unitary representations of the symmetry group that implements the principle of relativity for the relevant spacetime. Thus, Galilean quantum mechanics is grounded on the unitary projective representations of the Galilei group while Lorentzian quantum mechanics, including quantum field theory, is grounded on the unitary representations of the Poincaré group. Quantum mechanics and quantum field theory on a deSitter spacetime may also be viewed as being grounded on the unitary representations of the deSitter group. Both Galilei and Poincaré groups involve transformations amongst only inertial reference frames, the privileged class of reference frames with respect to which accelerations of bodies can be understood as the result of forces impressed upon them. Standard quantum mechanics, too, being grounded on the representations of either the Galilei group or the Poincaré group, is consequently meaningful as a physical theory with respect to this class of privileged reference frames. The primary goal of both [1,2] was to consider the symmetry group of transformations amongst non-inertial reference frames and extend the view that quantum physics is grounded on the principle of relativity to include non-inertial reference frames. In such a setting, a natural question that arises is the role of the equivalence principle and fictitious forces in quantum mechanics. As is well known, Einstein initially used accelerated systems and their accompanying fictitious forces as a guide to understanding the nature of gravitation in classical physics. Whether this also can be done for quantum theory will be investigated in this and subsequent papers.

Both [1,2] introduce the concept of the line group of the three dimensional Euclidean group, the group of (analytic) functions on the real line taking values on the Euclidean group, as the group of acceleration transformations. The main difference between the two papers is that in [2] time translations are included in the transformation group leading to a semidirect product of the Euclidean line group and the real line, called the Galilean line group, while in [1] time translations are not introduced explicitly and the analysis is carried out by way of transformation properties of the Schrödinger equation under the Euclidean line group. As a result, the representations of the two studies are different, particularly with regard to their cocycle structure. However, when the time translation parameter is set to zero, the representations of [2] reduce to those of [1] up to a coboundary (i.e., a removable phase factor). Both studies come to similar conclusions:

1. A unitary irreducible cocycle representation of acceleration transformations can be constructed on the same Hilbert space that carries a given unitary irreducible projective representation of the Galilei group.
2. The Hamiltonian for such a representation, when compared to the Hamiltonian that follows from the Galilei group, has an additive term that depends on the acceleration. Moreover, this term is proportional to the inertial mass, just as one would expect from the classical equivalence principle.
3. These properties lead to the natural interpretation of the term of the Hamiltonian that arises from the non-inertial nature of a reference frame as a "fictitious potential energy" term. To the extent the equality of inertial and gravitational masses holds, this fictitious potential energy term in turn has interpretation as gravitational potential energy, obtained now as a fundamental quantum mechanical entity (as opposed to, say, "quantizing" a pre-existing classical field).
4. Gravitational fields obtainable this way have zero spatial gradient (potential energy linear in position), though they may have arbitrary time dependence. This is the main limitation of [1,2], although it is not surprising that acceleration transformations that are functions of time alone give rise to gravitational fields that are functions of time alone.