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Finite field-dependent symmetries in perturbative quantum gravity

Sudhaker Upadhyay

S. N. Bose National Centre for Basic Sciences, Block JD, Sector III, Salt Lake, Kolkata -700098, India

HIGHLIGHTS

- The perturbative quantum gravity is treated as gauge theory.
- BRST and anti-BRST transformations are developed in linear and non-linear gauges.
- BRST transformation is generalized by making it finite and field dependent.
- Connection between linear and non-linear gauges is established.
- Using BV formulation the results are established at quantum level also.

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ABSTRACT

In this paper we discuss the absolutely anticommuting nilpotent symmetries for perturbative quantum gravity in general curved spacetime in linear and non-linear gauges. Further, we analyze the finite field-dependent BRST (FFBRST) transformation for perturbative quantum gravity in general curved spacetime. The FFBRST transformation changes the gauge-fixing and ghost parts of the perturbative quantum gravity within functional integration. However, the operation of such symmetry transformation on the generating functional of perturbative quantum gravity does not affect the theory on physical ground. The FFBRST transformation with appropriate choices of finite BRST parameter connects non-linear Curci–Ferrari and Landau gauges of perturbative quantum gravity. The validity of the results is also established at quantum level using Batalin–Vilkovisky (BV) formulation.

1. Introduction

The study of the structure of spacetime at Planck scale, where the quantum effects of gravity cannot be neglected, is a great challenge for fundamental physics. It is very essential to understand the

E-mail addresses: sudhaker@boson.bose.res.in, sudhakerupadhyay@gmail.com.

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perturbative quantum gravity for those who want to proceed towards any kind of non-perturbative approach [1]. The perturbative quantum gravity in curved spacetime as a gauge theory is a subject of interest form many respects [2–4]. The mode analysis and Ward identities for a ghost propagator for perturbative quantum gravity in de Sitter space has been discussed iteratively [5,6]. The Feynman rules and propagator for gravity in the presence of a flat Robertson–Walker background in the physically interesting cases of inflation have been analysed [7]. Such models of gravity have founded great attempts to unify gravity with Maxwell theory [8]. The gravity models with gauge invariance have their relevance in string theories also [9–11].

The quantum theory of gravity in general curved spacetime has general coordinate (gauge) invariance and hence cannot be quantized without getting rid of the redundant degrees of freedom. This can be achieved by imposing a suitable gauge conditions. The Landau and non-linear Curci–Ferrari gauge conditions play a pivotal role in the analysis of gauge and ghost condensation of the perturbation theory [12,13]. These gauge conditions can be incorporated to the theory of gravity at quantum level by adding the suitable gauge-fixing and ghost terms to the classical action which remains invariant under the fermionic rigid BRST invariance [14,15]. The BRST symmetry plays an important role to study the unitarity and renormalizability of the gauge theories [16,17]. However, BV formulation to quantize the more general gauge theories with open gauge algebra is a more fundamental approach to study the supergravity and topological field theories [16–22]. The BRST and the anti-BRST symmetries for perturbative quantum gravity in flat spacetime dimensions have been studied by many people [23–25] and their work has been summarized by N. Nakanishi and I. Ojima [26]. The BRST symmetry in two dimensional curved spacetime has been thoroughly studied [27–29]. Recently, the BRST formulation in the theory of perturbative quantum gravity has been analysed [30,31]. The BRST symmetry transformations of the gauge theories in flat spacetime have been generalized by making the parameter finite and field-dependent which is known as FFBRST transformations [32]. The FFBRST transformations have found several applications in gauge field theories in flat spacetime [32–39]. However, so far FFBRST formulation has not been developed for any theory of curved spacetime. This provides a motivation to develop FFBRST transformation in curved spacetime. We develop such a formulation for the first time for the theory of quantum gravity in the curved spacetime.

In this paper we discuss the BRST and anti-BRST invariance of gravity theory in Landau and massless Curci–Ferrari gauges. Further, we investigate the FFBRST transformation for perturbative quantum gravity. The FFBRST transformation is constructed by replacing the infinitesimal field-independent BRST parameter with a finite field-dependent global parameter. The formal aspects of such FFBRST formulation are discussed with full generality, in which we show that the FFBRST transformation is symmetry of the action, however, it does not leave the path integral measure of functional integral invariant. The explicit form of the non-trivial Jacobian of the path integral measure is calculated for the theory of quantum gravity. The non-trivial Jacobian changes the gauge-fixing and ghost terms within the functional integral of perturbative quantum gravity. We explicitly show that for a proper choice of field-dependent parameter the FFBRST transformation connects the linear and non-linear gauges within the functional integration of perturbative quantum gravity. The results are also tested at quantum level using BV formulation of perturbative quantum gravity.

This paper is organized as follows. In Section 2, we discuss the different gauge conditions in perturbative gravity with BRST invariance. In Section 3, we develop the field-dependent BRST symmetry in the theory of curved spacetime and show that Landau and non-linear gauges can be connected with suitable choices of finite parameter. The result is also established at quantum level in Section 4. In the last section, we summarize the results with some discussion on future investigations.

2. The perturbative quantum gravity

We start with the classical Lagrangian density for gravity in general curved spacetime

$$\mathcal{L}_c = \frac{\sqrt{-g}}{16\pi G}(R - 2\lambda), \tag{1}$$

where R is Ricci scalar curvature and λ is a cosmological constant.