Annals of Physics 340 (2014) 119–170

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Color Glass Condensate in Schwinger–Keldysh QCD

Sangyong Jeon *

Department of Physics, McGill University, 3600 University Street, Montréal QC, H3A-2T8, Canada

HIGHLIGHTS

- Application of the Schwinger–Keldysh formalism to many-body QCD.
- Clean separation of classical and quantum degrees of freedom.
- Identification of the correct coupling between the gluon field and the color source.
- Identification of the correct gauge transformation rules.
- Sources of the classicality and quantum corrections to JIMWLK clarified.

ARTICLE INFO

Article history:
Received 12 August 2013
Accepted 28 September 2013
Available online 8 October 2013

Keywords:
Color Glass Condensate
Relativistic heavy ion collisions
Many-body QCD
JIMWLK equation

ABSTRACT

Within the Schwinger–Keldysh representation of many-body QCD, it is shown that the leading quantum corrections to the strong classical color field are “classical” in the sense that the fluctuation field still obeys the classical Jacobi-field equation, while the quantum effects solely reside in the fluctuations of the initial field configurations. Within this context, a systematic derivation of the JIMWLK renormalization group equation is presented. A clear identification of the correct form of gauge transformation rules and the correct form of the matter-field Lagrangian in the Schwinger–Keldysh QCD is also presented.

1. Introduction

The study of small x gluons in a heavy nucleus in terms of classical gluon fields was initiated by L. McLerran, R. Venugopalan, A. Ayala and J. Jalilian-Marian in a series of seminal papers [1–5]. Since then the central idea called Color Glass Condensate (CGC) and its generalization to nucleus–nucleus

* Tel.: +1 5143986516.
E-mail address: jeon@physics.mcgill.ca.

0003-4916/$ – see front matter
http://dx.doi.org/10.1016/j.aop.2013.09.019

interaction, the Glasma, have inspired much work among theoreticians and experimentalists alike. Good reviews can be found in Refs. [6–15].

As CGC posits an ensemble of strong color charges producing a strong gluon field, its formulation properly belongs to the realm of many-body quantum field theory. As discussed in the following, the most natural language of many-body quantum field theory is the Schwinger–Keldysh (SK) formalism [16–21] (also known as the in–in formalism or the closed-time-path (CTP) formalism).

The main purpose of the present paper is to re-derive the well known JIMWLK renormalization group[1] equation [22–28] using the Schwinger–Keldysh formalism in a systematic way. The JIMWLK renormalization group equation is a non-linear generalization of the BFKL (Balitsky–Fadin–Kuraev–Lipatov) renormalization group equation [29,30]. These equations evolve the unintegrated gluon distribution function (also known as the transverse momentum dependent (TMD) gluon distribution function) towards small x region resumming $(\alpha_S \ln(1/x))^n$ contributions and were successful in explaining the results from HERA experiments [31–34].

The JIMWLK equation incorporates recombination processes leading to the gluon saturation [35,36] which becomes increasingly important as the density of small x gluons becomes higher in the ultrarelativistic limit. Along the way, we identify where this study differs from previous approaches and also where NNLO contributions should appear. This paper thus presents the proof-of-principle calculations which set up the stage for the more elaborate NNLO calculations.

In Ref. [37], Iancu, McLerran and Leonidov first introduced the Schwinger–Keldysh formalism for CGC and JIMWLK. Subsequently, F. Gelis, R. Venugopalan and their collaborators developed the diagrammatic approach [38,39] which was later used in many applications [40–48] such as particle productions and approach to thermalization, and factorizations. Yet, there are many benefits of using the Schwinger–Keldysh closed-time path integral explicitly in contrast to the diagrammatic approach. The main benefit exploited in this paper is the clear and clean separation of strong classical degree of freedom and the quantum degree of freedom. This is well known in condensed matter physics (for instance, see Refs. [49,50]), and similar conclusion was reached within the 2-PI effective action approach as well [51–55]. However, this benefit appears to have not been widely appreciated in the CGC context.

One of the main conclusions of this paper is that the leading order quantum corrections to the strong classical field come solely from the quantum fluctuations in the initial condition while the field itself still obeys the classical field equation. This makes it particularly simple to resum the leading log divergences (the JIMWLK equation) and the secular divergences [38,39] as those terms arise only from such quantum corrections. It also enables a clear derivation of the retarded, advanced and symmetric propagators in the classical background field and where they should appear in any expression for a diagram.

The separation of the strong classical degrees of freedom and quantum corrections is actually rather easy to see within the Schwinger–Keldysh formalism, especially in the r–a, or the Keldysh, representation. Consider for simplicity a scalar field theory. In the usual closed-time-path formulation, the time contour starts from the initial time, goes forward to the final time and comes back to the initial time. In the r–a representation, the field on the forward time line (call it ϕ_1) and the backward time line (call it ϕ_2) are combined as the common part $\phi_r = (\phi_1 + \phi_2)/2$ and the difference part $\phi_a = \phi_1 - \phi_2$. As will be discussed shortly in Section 2, the generating functional in the r–a representation is then given by [56,57]

$$\mathcal{Z}[J_r, J_a] = \int \mathcal{D}\phi_r \mathcal{D}\phi_a \rho_v[\phi_r^i, \dot{\phi}_r^i] \exp\left[i \int \left(\phi_a E[\phi_r, J_a] + J_r \phi_r - \frac{1}{24}\phi_a^3 V'''(\phi_r) \right) \right] \tag{1.1}$$

where

$$E[\phi_r, J_a] = \left(-\partial^2 \phi_r - m^2 \phi_r - V'(\phi_r) + J_a \right) \tag{1.2}$$

[1] JIMWLK stands for J. Jalilian-Marian, E. Iancu, L. McLerran, H. Weigert, A. Leonidov and A. Kovner.