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Optical analogue of relativistic Dirac solitons in binary waveguide arrays

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HIGHLIGHTS

- An optical analogue of Dirac solitons in nonlinear binary waveguide arrays is suggested.
- Analytical solutions to pseudo-relativistic solitons are presented.
- A correspondence of optical coupled-mode equations with the nonlinear relativistic Dirac equation is established.

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ABSTRACT

We study analytically and numerically an optical analogue of Dirac solitons in binary waveguide arrays in the presence of Kerr nonlinearity. Pseudo-relativistic soliton solutions of the coupled-mode equations describing dynamics in the array are analytically derived. We demonstrate that with the found soliton solutions, the coupled mode equations can be converted into the nonlinear relativistic 1D Dirac equation. This paves the way for using binary waveguide arrays as a classical simulator of quantum nonlinear effects arising from the Dirac equation, something that is thought to be impossible to achieve in conventional (i.e. linear) quantum field theory.

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1. Introduction

Waveguide arrays have been used intensively to simulate the evolution of nonrelativistic quantum mechanical particles in a periodic potential. Many fundamental phenomena in nonrelativistic classical and quantum mechanics, such as Bloch oscillations [1,2], Zener tunneling [3,4], optical dynamical localization [5], and Anderson localization in disordered lattices [6] have been simulated both theoretically and experimentally with waveguide arrays. In recent studies it was shown that, rather surprisingly, most of nonlinear fiber optics features (such as resonant radiation and soliton self-wavenumber shift) can also take place in specially excited arrays [7,8]. Recently, binary waveguide arrays (BWAs) have also been used to mimic relativistic phenomena typical of quantum field theory, such as Klein tunneling [9,10], the *Zitterbewegung* (trembling motion of a free Dirac electron) [11,12], and fermion pair production [13], which are all based on the properties of the Dirac equation [14]. Although there is as yet no evidence for fundamental quantum nonlinearities, nonlinear versions of the Dirac equation have been studied for a long time. One of the earlier extensions was provided by Heisenberg [15] in the context of field theory and was motivated by the question of mass. In the quantum mechanical context, nonlinear Dirac equations have been used as effective theories in atomic, nuclear and gravitational physics [16–19] and, more recently, in the study of ultracold atoms [20,21]. In this regard, BWAs can offer a rather unique model system to simulate nonlinear extensions of the Dirac equation when probed at high light intensities. The discrete gap solitons in BWAs in the *classical* context have been investigated both numerically [22–24] and experimentally [25]. In particular, in Ref. [23] soliton profiles with even and odd symmetry were numerically calculated and a scheme with two Gaussian beams, which are tuned to the Bragg angle with opposite inclinations, was proposed to efficiently generate gap solitons. In Ref. [25] solitons were experimentally observed when the inclination angle of an input beam is slightly above the Bragg angle.

Inspired by the importance of BWAs as a classical simulator for relativistic quantum phenomena, and also by past achievements in the investigation of discrete gap solitons in BWAs, in this work we present analytical soliton solutions of the discrete coupled-mode equations (CMEs) for a BWA and construct Dirac solitons of a nonlinear relativistic 1D Dirac equation in the quasicontinuous limit. This paves the way for using BWAs to simulate nonlinear extensions of the Dirac equation that violate Lorentz invariance [26], as well as other solitonic and nonsolitonic effects of nonlinear Dirac equations.

2. Analytical soliton solutions

Light propagation in a discrete, periodic binary array of Kerr nonlinear waveguides can be described, in the continuous-wave regime (CW), by the following dimensionless CMEs [9,22]:

$$i\frac{da_n(z)}{dz} = -\kappa[a_{n+1}(z) + a_{n-1}(z)] + (-1)^n\sigma a_n - \gamma|a_n(z)|^2 a_n(z), \tag{1}$$

where a_n is the electric field amplitude in the nth waveguide, z is the longitudinal spatial coordinate, 2σ and κ are the propagation mismatch and the coupling coefficient between two adjacent waveguides of the array, respectively, and γ is the nonlinear coefficient of waveguides, which is positive for self-focusing, but negative for self-defocusing media. For simplicity, here we suppose all waveguides have the same nonlinear coefficient, but even if these nonlinear coefficients are different (provided they are comparable), then analytical soliton solutions shown later will not be changed, because as explained later, one soliton component is much weaker than both unity and the other component, and thus one can eliminate the nonlinear term associated with this weak soliton component. In the dimensionless form, in general, one can normalize variables in the above equation such that γ and κ are equal to unity. However, throughout this work we will keep these parameters explicitly in Eq. (1). Before proceeding further, it is helpful to analyze the general properties of the general solutions of Eq. (1). First of all, let us assume that $(a_{2n}, a_{2n-1})^T = i^{2n}(\varphi_{2n}, \varphi_{2n-1})^T$ is one solution of Eq. (1) with φ_{2n} and φ_{2n-1} being appropriate functions. In this case, if we change the sign of γ while keeping the other two parameters constant, one can easily show that a new solution of Eq. (1) will be $(a_{2n}, a_{2n-1})^T = i^{2n}(\varphi^*_{2n-1}, \varphi^*_{2n})^T$, where $*$ denotes the complex conjugation. Secondly, if the sign of σ is changed