Annals of Physics 338 (2013) 283–301

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Extremely correlated Fermi liquids in the limit of infinite dimensions

Edward Perepelitsky *, B. Sriram Shastry

Physics Department, University of California, Santa Cruz, CA 95064, USA

HIGHLIGHTS

- Infinite-dimensional t–J model ($J = 0$) studied within new ECFL theory.
- Mapping to the infinite U Anderson model with self consistent hybridization.
- Single particle Green's function determined by two local self energies.
- Partial projection through control variable λ.
- Expansion carried out to $O(\lambda^2)$ explicitly.

ARTICLE INFO

Article history:
Received 6 September 2013
Accepted 17 September 2013
Available online 25 September 2013

Keywords:
High dimensions
t–J model
Hubbard model
Extremely Correlated Fermi Liquid model
Strongly correlated electrons

ABSTRACT

We study the infinite spatial dimensionality limit ($d \to \infty$) of the recently developed Extremely Correlated Fermi Liquid (ECFL) theory (Shastry 2011, 2013) [17,18] for the t–J model at $J = 0$. We directly analyze the Schwinger equations of motion for the Gutzwiller projected (i.e. $U = \infty$) electron Green's function $\mathcal{G}$. From simplifications arising in this limit $d \to \infty$, we are able to make several exact statements about the theory. The ECFL Green's function is shown to have a momentum independent Dyson (Mori) self energy. For practical calculations we introduce a partial projection parameter λ, and obtain the complete set of ECFL integral equations to $O(\lambda^2)$. In a related publication (Zitko et al. 2013) [23], these equations are compared in detail with the dynamical mean field theory for the large U Hubbard model. Paralleling the well known mapping for the Hubbard model, we find that the infinite dimensional t–J model (with $J = 0$) can be mapped to the infinite-U Anderson impurity model with a self-consistently determined set of parameters. This mapping extends individually to the auxiliary Green's function $\mathbf{g}$ and the caparison

* Corresponding author. Tel.: +1 408 891 4906.
E-mail address: eperepel@ucsc.edu (E. Perepelitsky).

http://dx.doi.org/10.1016/j.aop.2013.09.010

factor μ. Additionally, the optical conductivity is shown to be obtainable from $\mathcal{G}$ with negligibly small vertex corrections. These results are shown to hold to each order in λ.

1. Introduction

1.1. Motivation

The Hubbard model (HM) with the Hamiltonian:

$$H = -\sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \boldsymbol{\mu} \sum_i n_i, \tag{1}$$

has attracted great theoretical interest in condensed matter physics, and is also a fairly realistic model of strongly correlated materials such as the cuprates. While the small $\frac{U}{t}$ limit is well described by standard Fermi-Liquid theory [1,2], the large and intermediate $\frac{U}{t}$ (strongly correlated) cases are much less well understood. Considerable progress has been made by considering the HM in the limit of infinite dimensions [3–11]. One important result is that the Dyson self energy, defined by inverting the expression for the electron Green's function $\mathcal{G}$:

$$\mathcal{G}(k) = \frac{1}{i\omega_k + \boldsymbol{\mu} - \epsilon_k - \Sigma_D(k)}, \tag{2}$$

becomes momentum independent in this limit [3–6]. Two other important results are the self-consistent mapping of the infinite dimensional HM onto the Anderson Impurity model (AIM), detailed in [8] (Dynamical Mean Field Theory), and the vanishing of the vertex corrections in the optical conductivity [10,11], so that the two particle response is obtainable from the single particle Green's function. The Dynamical Mean Field Theory (DMFT) provides a means for doing reliable numerical calculations for the Hubbard model, at any value of U and has continued to provide new, and interesting results [12,13].

A different approach to understanding strong correlations is to consider the extreme correlation limit, where on sets $U \to \infty$ at the outset. In this case, the Hilbert space is Gutzwiller projected so that only single occupancy is allowed on each lattice site. One such extremely correlated model, the t–J model, consists of taking the $U \to \infty$ limit of the Hubbard model (the t part of the model) and adding on a nearest neighbor anti-ferromagnetic coupling term (the J part of the model). The t model studied here, is obtained by dropping the J term and thus is identical to the $U = \infty$ limit of the HM. It has been argued by Anderson [14] that the t–J model describes the physics of the cuprates, thereby providing an impetus for its detailed study. The Hamiltonian for this model can be written in terms of the Hubbard X operators as [15]

$$H = -\sum_{ij\sigma} t_{ij} X_i^{\sigma 0} X_j^{0\sigma} - \boldsymbol{\mu} \sum_{i\sigma} X_i^{\sigma\sigma} + \frac{1}{2}\sum_{ij\sigma} J_{ij} X_i^{\sigma\sigma} + \frac{1}{4}\sum_{ij\sigma_1\sigma_2} J_{ij}\{X_i^{\sigma_1\sigma_2} X_j^{\sigma_2\sigma_1} - X_i^{\sigma_1\sigma_1} X_j^{\sigma_2\sigma_2}\}. \tag{3}$$

The operator $X_i^{ab} = |a\rangle\langle b|$ takes the electron at site i from the state $|b\rangle$ to the state $|a\rangle$, where $|a\rangle$ and $|b\rangle$ are one of the three allowed states $|\uparrow\rangle$, $|\downarrow\rangle$, or $|-\rangle$. Our present goal is to obtain a formally exact solution of the above t model in the limit of large dimensions by studying its equations of motion. This is designed to be methodologically independent of the available DMFT solution of the HM with $U = \infty$, and can be compared with it.

Our object of study is the Green's function written as

$$\mathcal{G}_{\sigma_1\sigma_2}(i,f) = -\langle T_\tau X_i^{0\sigma_1}(\tau_i) X_f^{\sigma_2 0}(\tau_f)\rangle, \tag{4}$$