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Entropy evolution law in a laser process ☆

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HIGHLIGHTS

- We apply the thermo-field dynamics method to the master equation of a laser.
- We find the operator-sum (Kraus) representation for the density operator.
- We find both the normally ordered and compact forms of $\rho(t)$ for $\rho_0 = |z\rangle\langle z|$.
- We find the exact expression of the laser's entropy.
- Our results reveal quantitatively how a laser beam can be generated in a laser.

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ABSTRACT

For the first time, we obtain the entropy variation law in a laser process after finding the Kraus operator of the master equation describing the laser process with the use of the entangled state representation. The behavior of entropy is determined by the competition of the gain and damping in the laser process. The evolution formula for the number of photons is also obtained.

1. Introduction

Since the theoretical foundation proposed by Albert Einstein in 1917 [1] and the building of first functioning laser by Theodore H. Maiman in 1960, lasers have been successfully applied in various areas, including the laser cooling technique developed by Chu et al. [2,3]. As one of the most important concept in physics, entropy measures the disorder of a system. Studying the evolution of the entropy, we can get a clear understanding of how a laser beam is created by appropriate pumping. Some work

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has been done concerning the entropy exchange between a laser and its environment [4,5]. However, the evolution of entropy in a laser itself has not yet been studied. In this work, we shall derive the entropy evolution law of a laser process. Our results explain how the self-organization phenomenon happens in a laser.

In quantum optics theory, the time evolution of a laser in the lowest-order approximation can be described by the following master equation of the density operator [6–9]:

$$\frac{d\rho(t)}{dt} = g\left[2a^\dagger\rho(t)a - aa^\dagger\rho(t) - \rho(t)aa^\dagger\right] + \kappa\left[2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a\right], \tag{1}$$

where g and κ are the cavity gain and the loss, respectively, and $a^\dagger$ and a are the photon creation operator and the photon annihilation operator, respectively. It is also known that the evolution due to the interaction between a system and its environment can be ascribed to an evolution from the initial density operator ρ_0 to $\rho(t)$:

$$\rho(t) = \sum_{n=0}^{\infty} M_n\rho_0 M_n^\dagger. \tag{2}$$

Such an expression is named an operator-sum (Kraus) representation, and M_n is named the Kraus operator. So far as our knowledge is concerned, the entropy variation in a laser channel has never been reported. In this paper, we shall show how the entropy of an initial coherent state $\rho_0 = |z\rangle\langle z|$ (the fact that an n-photon distribution in a coherent state is a Poisson distribution exactly fits the measurement result of photon distribution in a laser light) varies in the laser process. Before doing this, we first derive the Kraus operator by solving the master equation (1).

Our way to do this is by introducing the two-mode entangled state

$$|\eta\rangle = \exp\left(-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^*\tilde{a}^\dagger + a^\dagger\tilde{a}^\dagger\right)|0\tilde{0}\rangle, \tag{3}$$

where $\tilde{a}^\dagger$ is a fictitious mode independent of the real mode $a^\dagger$, and $|\tilde{0}\rangle$ is annihilated by $\tilde{a}$, $\left[\tilde{a}, \tilde{a}^\dagger\right] = 1$. The state $|\eta = 0\rangle$ possesses the properties

$$\begin{aligned} a|\eta = 0\rangle &= \tilde{a}^\dagger|\eta = 0\rangle, \\ a^\dagger|\eta = 0\rangle &= \tilde{a}|\eta = 0\rangle, \\ (a^\dagger a)^n|\eta = 0\rangle &= (\tilde{a}^\dagger\tilde{a})^n|\eta = 0\rangle. \end{aligned} \tag{4}$$

Operating both sides of (1) on the state $|\eta = 0\rangle \equiv |I\rangle$, and denoting $|\rho\rangle = \rho|I\rangle$, and using (4), we have the time-evolution equation for $|\rho(t)\rangle$:

$$\frac{d}{dt}|\rho(t)\rangle = \begin{bmatrix} g\left(2a^\dagger\tilde{a}^\dagger - aa^\dagger - \tilde{a}\tilde{a}^\dagger\right) \\ +\kappa\left(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a}\right) \end{bmatrix}|\rho(t)\rangle, \tag{5}$$

where $|\rho_0\rangle \equiv \rho_0|I\rangle$, and ρ_0 is the initial density operator.

The formal solution of (5) is

$$|\rho(t)\rangle = U(t)|\rho_0\rangle, \tag{6}$$

and

$$U(t) = \exp\begin{bmatrix} gt\left(2a^\dagger\widetilde{a}^\dagger - aa^\dagger - \widetilde{a}\widetilde{a}^\dagger\right) \\ +\kappa t\left(2a\widetilde{a} - a^\dagger a - \widetilde{a}^\dagger\widetilde{a}\right) \end{bmatrix}. \tag{7}$$

It is challenging to disentangle the exponential operator $U(t)$. This reminds us of two theorems about the normally ordered expansion of multimode bosonic exponential operators, which is helpful in disentangling $U(t)$.