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Bogoliubov transformations and fermion condensates in lattice field theories

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ABSTRACT

We apply generalized Bogoliubov transformations to the transfer matrix of relativistic field theories regularized on a lattice. We derive the conditions these transformations must satisfy to factorize the transfer matrix into two terms which propagate fermions and antifermions separately, and we solve the relative equations under some conditions. We relate these equations to the saddle point approximation of a recent bosonization method and to the Foldy–Wouthuysen transformations which separate positive from negative energy states in the Dirac Hamiltonian.

1. Introduction

In this paper we investigate some properties of Bogoliubov transformations in fermionic lattice field theories in connection with the appearance of fermionic condensates. We are motivated by our study of a new bosonization method [1], but our results have a wider relevance, and relate these transformations to the ones introduced by Foldy–Wouthuysen to separate fermions from antifermions in the Dirac Hamiltonian [2].

Bogoliubov transformations are unitary transformations which mix creation and annihilation operators. They have been introduced in the theory of many-body systems in which have been extensively used for their simplicity, in particular in connection with variational principles, and also as a starting point of more elaborated approximation methods. In their application to superconductivity they

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reproduce the results of the BCS theory of electron–electron interactions with an easy extension to the case of electrons interacting with phonons [3]. While mixing of creation-annihilation operators was a novelty in the theory of many-body systems, mixing of particles and antiparticles in the Hamiltonian formalism of relativistic field theories is very natural, and in fact generalized Bogoliubov transformations have been used also in this domain. See for example [4] where general bilinear fermionic Hamiltonians are diagonalized, or more recent studies of QCD in the limit of large number of colours [5]. Bogoliubov transformations are a fundamental tool in understanding the black body radiation in the Unruh effect in accelerating reference frame [6] and the black hole Hawking radiation [7]. But the specific difficulties of the renormalization procedure in the Hamiltonanian formalism have limited their use in quantum field theory.

Recently Bogoliubov transformations have found a somewhat different application in lattice field theory in the formalism of the transfer matrix, which is close from the physical point of view to the Hamiltonian formalism. They have been used to introduce dynamical composite bosons in fermionic theories. An independent Bogoliubov transformation at each time slice can be performed in the operator form of the partition function. The time-dependent parameters of the transformation are then associated with composite bosonic fields in the presence of fermionic fields (quasi-particles) satisfying a compositeness condition which avoids double counting [8,9]. One thus gets an effective action of composite fields plus quasi-particles, exactly equivalent to the original one, in which ground state and excited states can be treated on the same footing. Of course, in practical applications, some approximation must be introduced.

A Bogoliubov transformation generates a new vacuum which has the form of a fermion condensate. We studied such a condensate in a first approach to bosonization [1], which can be regarded as an approximation of the method of Refs. [8,9], in which quasi-particles are altogether neglected. In an application to a four-fermion interaction model with a discrete chiral symmetry (at zero fermion mass), an explicit form of the fermion condensate appearing when the symmetry is spontaneously broken was found in a saddle point approximation. The condensate is made of a composite boson which is a superposition of a symmetry breaking plus a symmetry conserving state. This result is not surprising from the point of view of the renormalization group, because it tells that we have a contribution of two operators of the same dimension which are no longer separated by symmetry. But looking carefully at this result we found that condensation of a symmetry conserving boson takes place also in the free theory, if we require factorization of the transfer matrix in two terms which propagate particles and antiparticles separately. This requirement gives rise to the same equations as the requirement of extremality of the vacuum energy which is generated by the Bogoliubov transformation, in the same way as in the many-body theory [3]. The saddle point approximation of the effective action we are talking about, under the assumption that the saddle point equations have stationary solutions, equals the effective action obtained after a time independent Bogoliubov transformation by neglecting quasi-particles, without any reference to the bosonization method. We will present our results in this more general perspective, but keeping in mind that, by time independent Bogoliubov transformations, we are investigating the saddle point stationary solutions of the mentioned approximated bosonization method.

At this point the relation of Bogoliubov transformations with Foldy–Wouthuysen transformations should be clear. The latter ones eliminate the mixing between positive and negative energy solutions in the continuum free Dirac Hamiltonian. The explicit form of these transformations has also been found for some interactions, which include minimal [10] and anomalous [11] interaction of spin-1/2 particles with a time-independent magnetic field, anomalous interactions with a time-independent electric field and with a pseudo-scalar field [12]. Other generalizations are discussed in [13]. See also [14] and references therein. In the second quantization formalism the separation of positive- from negative-energy states is accompanied by the generation of a new vacuum. Of course in the free-field case this vacuum will be unitary-equivalent to the previous one, but more intricate possibilities are opened in the interacting case in presence of a phase transition. We found also interesting to see how, in the correspondence from the first and second quantization, the ambiguities in the Foldy–Wouthuysen transformation [11] appear as multiple solutions of the saddle-point equations for the corresponding Bogoliubov transformation, and they are solved by demanding that the new vacuum energy be minimal.