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ScienceDirect
Annals of Physics 323 (2008) 2286–2310
ANNALS of PHYSICS
www.elsevier.com/locate/aop

Spectrum of the non-abelian phase in Kitaev's honeycomb lattice model

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Received 7 December 2007; accepted 16 December 2007
Available online 3 January 2008

Abstract

The spectral properties of Kitaev's honeycomb lattice model are investigated both analytically and numerically with the focus on the non-abelian phase of the model. After summarizing the fermionization technique which maps spins into free Majorana fermions, we evaluate the spectrum of sparse vortex configurations and derive the interaction between two vortices as a function of their separation. We consider the effect vortices can have on the fermionic spectrum as well as on the phase transition between the abelian and non-abelian phases. We explicitly demonstrate the 2^n-fold ground state degeneracy in the presence of $2n$ well separated vortices and the lifting of the degeneracy due to their short-range interactions. The calculations are performed on an infinite lattice. In addition to the analytic treatment, a numerical study of finite size systems is performed which is in exact agreement with the theoretical considerations. The general spectral properties of the non-abelian phase are considered for various finite toroidal systems.

PACS: 05.30.Pr; 75.10.Jm

Keywords: Topological models; Non-abelian vortices; Kitaev's model

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doi:10.1016/j.aop.2007.12.009

1. Introduction

Topological quantum computation [1–4] is certainly among the most exotic proposals for performing fault-tolerant quantum information processing. This approach has attracted considerable interest, since it is closely related to the problem of classifying topologically ordered phases in various condensed matter systems. The connection is provided by anyonic quasiparticles, which appear as states of topologically ordered systems with non-trivial statistical properties. Some of these anyon models can support universal quantum computation. Up to now, no complete classification of topological phases exists in terms of their physical properties or their computational power. This is due to the small number of analytically treatable models that exhibit topological behavior. The most studied arena is the celebrated fractional Quantum Hall effect [5,6] appearing in a two-dimensional electron gas when it is subject to a perpendicular magnetic field.

Recently various two-dimensional lattice models exhibiting topological behavior have been proposed [7–13] that enjoy analytic tractability. One such lattice proposal is the honeycomb model introduced by Alexei Kitaev [10]. It consists of a two-dimensional honeycomb lattice with spins at its vertices subject to highly anisotropic spin–spin interactions. This model has several remarkable features. It is exactly solvable and can thus be studied analytically. For particular values of the couplings, the model can be mapped to Z_2 gauge theory on a square lattice (the toric code), which supports abelian anyons. This anyon model has been employed for performing various quantum information tasks [2]. When one adds an external magnetic field, the model supports non-abelian Ising anyons. Even though neither model supports universal quantum computation, particular variations of the latter have been considered for this purpose [14,15]. One expects that when the couplings of the honeycomb lattice model are varied, the system will undergo a phase transition between the abelian and non-abelian phases. The existence of the different phases is only argued in the original work [10] based on mathematical considerations and no rigorous presentation of the transition is provided.

So far, the studies on Kitaev's honeycomb lattice model have concentrated on the abelian phase [16–18]. Here we present an extensive study of its spectral properties in the presence of an external magnetic field. Solving the model for various sparse vortex configurations gives us qualitative and quantitative results for the behavior of the spectrum in the non-abelian phase. The study includes the explicit demonstration of zero modes in the presence of well separated vortices and the lifting of the degeneracy due to their short-range interaction. These properties are subsequently connected to the properties of the Ising anyon model giving direct evidence that the low energy behavior of the non-abelian phase is indeed captured by this model. In addition, we consider the stability of the different phases, which is of importance when one is interested in physically realizing the model [19]. The analytic calculations are supported by exact numeric diagonalizations of finite size systems. The exact agreement between the analytic and numeric solutions for these finite size systems is demonstrated and the effect of an external effective magnetic field on finite size systems is discussed. Our work generalizes the analysis in [17] performed by one of the authors, where only the abelian phases in the limiting vortex-free and full-vortex cases were considered.

The paper is organized as follows. In Section 2 we give an overview of the honeycomb lattice model. There we outline the analytic approach for solving the model for various vortex configurations by employing Majorana fermionization. Section 3 provides explic-