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Three-loop corrections in a covariant effective field theory

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Abstract

Chiral effective field theories have been used with success in the study of nuclear structure. It is of interest to systematically improve these energy functionals (particularly that of quantum hadrodynamics) through the inclusion of many-body correlations. One possible source of improvement is the loop expansion. Using the techniques of Infrared Regularization, the short-range, local dynamics at each order in the loops is absorbed into the parameterization of the underlying effective Lagrangian. The remaining nonlocal, exchange correlations must be calculated explicitly. Given that the interactions of quantum hadrodynamics are relatively soft, the loop expansion may be manageable or even perturbative in nuclear matter. This work investigates the role played by the three-loop contributions to the loop expansion for quantum hadrodynamics.

1. Introduction

Density Functional Theory (DFT) is a powerful technique originally developed for use in condensed matter physics [1–3] that has been successfully adapted to nuclear physics [4–9]. DFT states that the ground-state expectation value of any observable is a *unique* functional of the exact ground-state density; moreover, if the expectation value of the Hamiltonian is considered as a functional of the density, then the exact ground-state density can be determined by minimizing the energy functional [1,3]. Furthermore, DFT

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allows one to replace the quantum many-body equations by a series of single-particle equations with local, classical fields that reproduce certain observables exactly (energy, scalar and vector densities, and chemical potential) [2,9,10]. Thus, the problem is reduced to determining the exact ground-state energy functional. However, this is impossible in practice. As a result, a number of approximate energy functionals have been developed; one such theory is based on quantum hadrodynamics (QHD).

QHD is a low-energy theory of the strong interaction [7–9,11–15]. Here the hadron, and not the quark, is the observed degree of freedom (due to confinement at this energy scale). QHD models the nuclear force as a exchange of mesons between nucleons. Isoscalar–scalar (σ) and vector (ω) mesons represent a medium-range attraction and a short-range repulsion, respectively. The pion is also included to take chiral symmetry into account. DFT allows one to replace the quantum meson fields with their classical equivalents (Kohn–Sham potentials called mean fields); these mean fields, while large, are small compared to the chiral symmetry breaking scale. As a result, one can use these ratios as small parameters with which to expand the energy functional in a controlled fashion. Each term in this Lagrangian is characterized by an undetermined coefficient which is assumed to be natural, or order unity [16,17]. The resulting Lagrangian [7,8] has provided a method for predicting the properties of nuclei [18–23].

However, the mean field theory of QHD is only an approximation to the exact energy functional. The question remains how this theory behaves when other many-body corrections are included to improve the energy functional (such as loops, rings, clustering, etc.). This work is part of an investigation into the effects of loops on this particular energy functional and is a continuation of [24,25]. In these works, the many-body loop expansion was carried out to the two-loop order where, using the techniques of Infrared Regularization, all of the short-range, local dynamics were absorbed into the parameterization of the underlying effective Lagrangian. What remained was the nonlocal exchange correlations, which were then explicitly calculated. In addition, the effect of this expansion on naturalness was investigated and the size of the two-loop exchange integrals was determined. Since the interactions of QHD are relatively soft, one might expect that, once Infrared Regularization is taken into account, the loop expansion may be asymptotic. It is the purpose of this work to investigate the effect of the three-loop contributions on this theory.

The loop expansion for QHD is constructed in the usual manner [26–32]. The effective action is expanded around its classical value by grouping terms according to the number of quantum loops (or powers of $\hbar$) in their corresponding diagrams. For the purposes of this work, we are interested only in the terms in the expansion at the three-loop level. All of the integrals that represent tadpole and disconnected diagrams cancel out in the effective action and we are left with only the fully connected diagrams. In addition, those diagrams which are anomalous are discarded. For the cases considered here, there are eleven integrals of interest. Ten of these integrals have four factors of the nucleon propagator and two meson propagators (either scalar, vector, or pion) each. There is one additional three-loop diagram with two factors of both the baryon and the pion propagator. In this work, no nonlinearities in the isoscalar mesons were included in the effective Lagrangian, as in [24]. In [25], the effect of these nonlinearities was explored at the two-loop level. As we are interested only in the general effects of the loop expansion at third order (and not an improved equation of state), these nonlinearities are not retained. The effect of the inclusion of nonlinear meson self-interactions on the three-loop integrals is left for future work.