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Modeling force transmission in granular materials

Modélisation de la transmission des forces dans les matériaux granulaires

Farhang Radjai [a,b,*]

[a] *LMGC, UMR 5509 CNRS – University of Montpellier, place Eugène-Bataillon, 34095 Montpellier, France*
[b] *MultiScale Material Science for Energy and Environment, UMI 3466 CNRS–MIT, CEE, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge 02139, USA*

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ABSTRACT

The probability density function of contact forces in granular materials has been extensively studied and modeled as an outstanding signature of granular microstructure. Arguing that particle environments play a fundamental role in force transmission, we analyze the effects of steric constraints with respect to force balance condition and show that each force may be considered as resulting from a balance between lower and larger forces in proportions that mainly depend on steric effects. This idea leads to a general model that predicts an analytical expression of force density with a single free parameter. This expression fits well our simulation data and generically predicts the exponential fall-off of strong forces, a small peak below the mean force and the non-zero probability of vanishingly small forces.

RÉSUMÉ

La densité de probabilité des forces de contact dans les matériaux granulaires représente une signature remarquable de la microstructure granulaire et, à ce titre, elle a fait l'objet de nombreuses études et d'efforts de modélisation. Nous allons analyser le rôle fondamental des environnementaux locaux des particules pour la transmission des forces et les effets des contraintes stériques par rapport à l'équilibre des forces. Cette analyse permet de montrer qu'une force de contact met en jeu des forces supérieures et inférieures à cette force dans des proportions qui sont contrôlées par les effets stériques. Cette idée simple conduit à un modèle général qui prédit une expression analytique de la densité des forces avec un seul paramètre libre. Ce paramètre coïncide avec le degré d'homogénéité des forces et peut dépendre de l'anisotropie du réseau des contacts ou des formes et distributions des tailles des particules. Cette expression ajuste bien les données numériques et prédit d'une manière générique la décroissance exponentielle des forces fortes, un petit pic en dessous de la force moyenne et une densité de probabilité non nulle pour les forces très petites.

* Correspondence to: LMGC, UMR 5509 CNRS – University of Montpellier, place Eugène-Bataillon, 34095 Montpellier, France.
E-mail address: franck.radjai@univ-montp2.fr.

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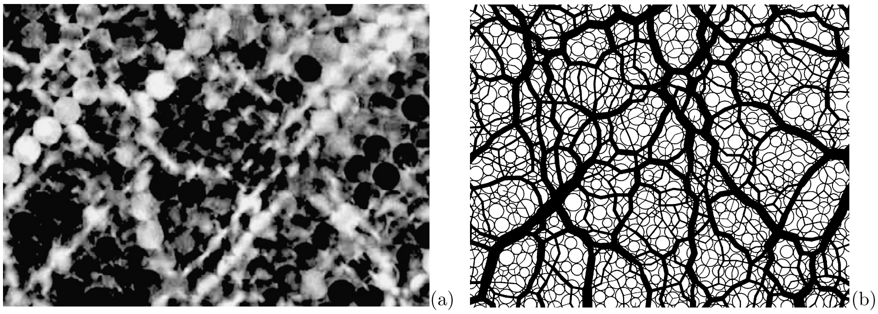

Fig. 1. (a) Photoelastic image of an assembly of birefringent spheres [18]. (b) Map of normal forces in a simulated packing of disks. Line thickness is proportional to normal force.

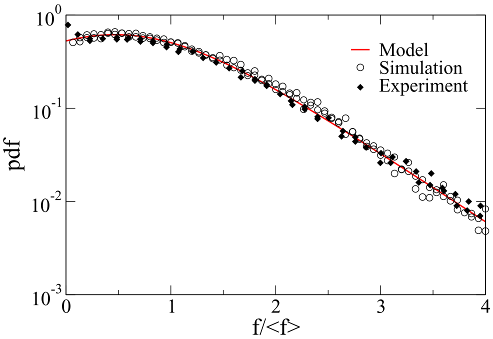

Fig. 2. (Color online.) Probability density function of normal contact forces normalized by the mean force in static granular materials from simulations and experimental data of Mueth et al. [8]. The solid line is the prediction of the model introduced in this paper yielding expression (10).

1. Introduction

Geometrical disorder and steric exclusions in granular materials lead to an unexpectedly inhomogeneous distribution of contact forces [1–13]. The inhomogeneous transmission of stresses was initially observed by means of photoelastic experiments with particles made of a birefringent material [1,2]. A photoelastic image, such as that displayed in Fig. 1(a), readily reveals filaments of bright grains subjected to high loads against a background of dark zones composed of much less loaded grains. Such filamentary patterns of stresses inside grains are induced by strong contact forces as those shown in Fig. 1(b) from numerical simulations, and are known as *force chains*. The linear aspect of chains observed in most reported experiments reflects long-range ordering of grains of nearly the same size in a 2D geometry. But this linear aspect of strong force chains is more generally observed in simulations of arbitrary grain shapes and size distributions in 2D and 3D, in which a variety of other patterns such as arch-type chains are quite common [10,14–17].

The forces were measured for the first time by using carbon paper to record normal force prints at the boundaries of a bead packing [4,8]. They were found to have a nearly uniform probability density function (pdf) in the range of weak forces followed by an exponential falloff of strong forces. Similar force distributions were found by later experiments [7,19] using contact area trace and by means of numerical simulations [5,6,20,10,11,21]. Fig. 2 shows a typical force density obtained by two different numerical methods for the same packing built by isotropic compaction as well as the data obtained by the experiments of Mueth et al. by means of carbon paper trace [8]. The data from our simulations and experiments coincide every where within the available precision in exception to the range of vanishingly small forces, where the details of sample preparation matter. The solid line is the plot of expression (10) predicted by the model that will be presented in this paper.

Detailed analysis of sheared granular materials provided further evidence for the bimodal organization of the force network in well-defined weak and strong networks with the strong network contributing almost exclusively to the shear strength and weak forces acting mainly to prop strong force chains [22]. All further investigations of force distributions have shown that the exponential distribution of strong forces is a robust feature of force distribution in granular media both in two and three dimensions. In contrast, the weak forces appear to be sensitive to the packing state resulting from the deformation history [21,23,24]. In an isotropic packing state, the distribution shows a relatively small peak below the mean force as seen in Fig. 2, and the probability density of small forces does not fall to zero [25,26]. The peak disappears in a sheared packing and the distribution of weak forces turns to a nearly decreasing power law [10,21,27]. This is also what generally is observed in frictional packings composed of aspherical grains or broad size distributions [28,15,17,29–31].

The sensitivity of force probability density to packing states and grain characteristics indicates that it can be used as structural descriptor of granular materials, and it might even be more relevant to rheology of granular materials than purely geometrical descriptors such as radial pair distributions [32]. In this sense, a successful modeling approach may reveal