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Slow granular flows: The dominant role of tiny fluctuations

Écoulements granulaires lents : Le rôle dominant des très petites fluctuations

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A B S T R A C T

What mechanism governs slow flows of granular media? Microscopically, the grains experience enduring frictional contacts in these flows. However, a straightforward translation to a macroscopic frictional rheology, where the shear stresses are proportional to the normal stresses with a rate-independent friction coefficient, fails to capture important aspects of slow granular flows. There is now overwhelming evidence that agitations, tiny fluctuations of the grain positions, associated with large fluctuation of their contact forces, play a central role for slow granular flows. These agitations are generated in flowing regions, but travel deep inside the quiescent zones, and lead to a nonlocal rheology.

R É S U M É

Quel est le mécanisme qui gouverne les écoulements lents d'un milieu granulaire ? À l'échelle microscopique, les grains sont soumis à des contacts frottants. Cependant, une interprétation directe de ces écoulements par une rhéologie macroscopique, dans laquelle les contraintes de cisaillement seraient proportionnelles aux contraintes normales avec un coefficient de frottement indépendant de la vitesse, ne permet pas de reproduire certaines propriétés importantes des écoulements lents. Il est maintenant clair que, lorsqu'elles sont associées à de grandes fluctuations des forces de contact, de très petites fluctuations des positions des grains jouent un rôle capital dans ces écoulements. Bien que l'agitation des grains provienne de zones en mouvement, elle pénètre profondément dans les régions inactives, donnant ainsi naissance à une rhéologie non locale.

1. Introduction & motivation

When the flow rate in a granular medium is so slow that the grains remain in contact most of the time, so that collisions play a minor role for the momentum transfer, we speak of slow or dense granular flows. Typical examples include the slow deformations of a granular pack under the motion of external boundaries, such as in Couette [1,2] and split-bottom [3]

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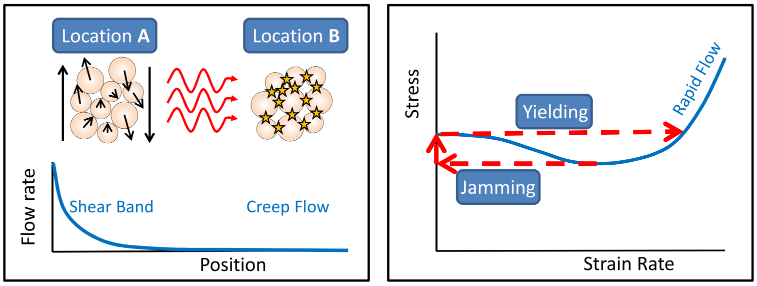

Fig. 1. (Color online.) (Left) Schematic depiction of the nonlocal "agitation" picture, where a localized flow in location A is associated with strong fluctuations in the microscopic motion. These lead to fluctuations in the contact forces that are carried through the medium (wavy lines) until they reach the nearly stationary zones (location B), where there is no appreciable grain motion, but the contact forces still fluctuate strongly (stars). (Right) Schematic depiction of a nonmonotonic flow curve. As a function of strain rate, the stresses initially decrease, before increasing sharply for rapid flows where collisions start to play a role. In controlled stress experiments, such a curve would lead to a hysteretic yielding-jamming behavior, as indicated.

geometries. We note that for faster flows, such as chute flows or avalanches, a successful description has been developed based on a rate-dependent friction coefficient [4–6]. In contrast, we still lack a predictive description for slow granular flows: given boundary conditions, what determines the flow rate and profile? How are stresses and strain rates related?

Slow granular flows differ in important ways from Newtonian flows, as friction plays a central role. This suggests that, first, shear stresses τ are proportional to the normal stresses P, and second, that the ratio of shear to compressive stress τ/P, which can be seen as an effective friction coefficient, μ_{eff}, does not vary strongly with rate and remains finite for strain rates going to zero [2,4–8].

These two assumptions form the core of a classical 'Mohr–Coulomb' picture of slow granular flows. First, they are consistent with rheological experiments such as in Couette, split-bottom or vane geometries, which find that the flow profiles do not vary with the driving rate, and the shear stresses are proportional to the confining pressure but rate independent [2,7–10]. Second, they give a simple picture for the yielding and jamming of granular flows: when τ/P is less than the effective friction coefficient of the medium, $\tau/P < \mu_{\text{eff}}$, the granular material remains essentially solid and does not flow, whereas flow sets in when $\tau/P > \mu_{\text{eff}}$—hence the frictional picture leads to a yielding criterion. This picture captures important aspects of the yielding of granular medium on an inclined plane: for small inclination angles θ, the sand remains fixed (here $\tau/P = \tan(\theta) < \mu_{\text{eff}}$), whereas for sufficiently large angles (where $\tau/P = \tan(\theta) \geq \mu_{\text{eff}}$), the sand starts to flow [4,6,8,11]. Third, this picture gives a simple explanation for the occurrence of sharp shear bands in granular flows, as small changes in the shear stress may lead to a drop from above to below the yielding criterion. For example, in Couette flows, the shear stress decays away from the inner cylinder with $1/r^2$ [12], leading to a narrow shear band near the inner wall, as observed experimentally [1].

However, a closer observation of these explanations, paired with a wealth of new experiments, illustrates important limitations of the Mohr–Coulomb picture. First, it ignores the crucial role played by fluctuations of the contact forces, which we will refer to as agitations, and second, such model cannot capture important aspects of the jamming/yielding transition. In the remainder of this paper, we will expand on these points (illustrated in Fig. 1), but before doing so, let us give the essential flavor of these.

Agitations: Let us consider the sharpness of the yielding criterion, which predicts the localization of flows in shear bands and a corresponding sharp separation between stationary, solid-like zones, and flowing zones [11,13–15]. There is by now overwhelming evidence against this picture: experiments clearly show that there is no sharp boundary between flowing and stationary zones, with (creep) flow occurring even far away from the main shear band. This makes the concept of shear band problematic; if one simply defines the shear band as all locations where the strain rate is finite, shear bands are very wide and often span the whole system. To distinguish the creep regions from the localized regions where most of the flow takes place, we will define shear bands as those regions where 99% of the strain is localized.

Moreover, several recent experiments [16–18] and theoretical works [19–22] indicate that for matter with granularity, the "fluidity" in location B, i.e. the local relation between stress and strain rate, is strongly influenced by the flow in location A. The physical picture is that localized flows lead to large fluctuations in the local contact forces, and that these fluctuations can propagate deep into the near-stationary zones. Even though the average shear stresses may be small there, the *fluctuations* in the contact forces lead to a finite probability that the local friction can be overcome. Consistent with this, the near-stationary or creep zones are observed to have a zero yield stress, and are thus not solid [14,17,18]. The resulting nonlocal behavior was first observed in the flow of emulsions [16], has also been observed in foams [23], and has been modeled by a diffusive model for the fluidity, leading to the introduction of a length scale that characterizes the nonlocality [16,20,21]. Kamrin and Koval have adapted these ideas for frictional rheologies, capturing granular Couette flows [19], and more recently, the full flow profiles and height dependence of split bottom granular flows [3,22,24,25].

Nonmonotonic friction: Let us now reconsider the assumption that μ_{eff} is constant at low rates (and eventually increases at rapid flow rates), which is motivated by the observed rate independence of the stresses for slow flows. Clearly, if the flow rate is not determined by the stresses, there is nothing that sets the flow rate for a given stress—this in itself is a major