C. R. Physique 16 (2015) 51–61

Contents lists available at ScienceDirect

Comptes Rendus Physique

www.sciencedirect.com

Granular physics / Physique des milieux granulaires

Kinetic theory for sheared granular flows

Théorie cinétique des écoulements granulaires cisaillés

Viswanathan Kumaran

Department of Chemical Engineering, Indian Institute of Science, Bangalore 560 012, India

ARTICLE INFO

Article history:
Available online 13 January 2015

Keywords:
Granular kinetic theory
Sheared granular flows
Dense granular flows

Mots-clés :
Théorie cinétique granulaire
Écoulements granulaires cisaillés
Écoulements granulaires denses

ABSTRACT

Rapid granular flows are far-from-equilibrium-driven dissipative systems where the interaction between the particles dissipates energy, and so a continuous supply of energy is required to agitate the particles and facilitate the rearrangement required for the flow. This is in contrast to flows of molecular fluids, which are usually close to equilibrium, where the molecules are agitated by thermal fluctuations. Sheared granular flows form a class of flows where the energy required for agitating the particles in the flowing state is provided by the mean shear. These flows have been studied using the methods of kinetic theory of gases, where the particles are treated in a manner similar to molecules in a molecular gas, and the interactions between particles are treated as instantaneous energy-dissipating binary collisions. The validity of the assumptions underlying kinetic theory, and their applicability to the idealistic case of dilute sheared granular flows are first discussed. The successes and challenges for applying kinetic theory for realistic dense sheared granular flows are then summarised.

RÉSUMÉ

Les écoulements granulaires rapides sont des systèmes forcés hautement dissipatifs hors équilibre, dans lesquels les interactions entre particules dissipent l'énergie, et qui, de ce fait, requièrent un apport énergétique ininterrompu pour agiter les particules et faciliter les réarrangements nécessaires à l'écoulement. Ceci les différencie des écoulements de fluides moléculaires, qui sont en général proches de l'équilibre, et dont les molécules sont agitées par des fluctuations thermiques. Les écoulements granulaires cisaillés constituent une classe dans laquelle l'énergie nécessaire est fournie par le cisaillement moyen. Ils sont étudiés à l'aide de la théorie cinétique des gaz, dans laquelle les particules sont traitées comme des molécules gazeuses, et où leurs intéractions sont binaires, instantanées et dissipatives. Nous discutons d'abord le bien-fondé de ces hypothèses qui sous-tendent la théorie cinétique et leur emploi dans le cas idéaliste d'un écoulement granulaire dilué. Nous résumons ensuite les succès et les défis attachés à la mise en œuvre de la théorie cinétique dans des écoulements réalistes denses et cisaillés.

© 2014 Académie des sciences. Published by Elsevier Masson SAS. All rights reserved.

E-mail address: kumaran@chemeng.iisc.ernet.in.

http://dx.doi.org/10.1016/j.crhy.2014.11.008
1631-0705/

1. Introduction

The physics of granular flows is a fascinating subject because a range of complex collective behaviour arise from seemingly simple microscopic interactions between the individual grains. The experiments of Faraday [1] on convection rolls in granular heaps was perhaps the first illustration of complex flows generated in granular media. Since then, there have been many studies on complex phenomena such as the formation of convection rolls [2], surface waves [3], patterns, and solitary waves [4]. Most of these complex phenomena can be reproduced in simulations using simple interactions at the grain level [5–7], where the grains are considered as soft or hard spheres. However, we currently lack a theoretical framework for starting from the microscopic particle interaction laws and deriving macroscopic equations. A relatively simpler phenomenon, which currently is not adequately explained on the basis of microscopic interaction laws, is the transition from a static to a flowing state when the angle of inclination of a granular material exceeds a critical value called the angle of repose. While the initiation of flow has been attributed to a 'yield stress' due to the friction between the grains in the static state, it is more difficult to explain the cessation of flow when the angle of inclination decreases below a critical value. In contrast, a horizontal layer of a Newtonian fluid flows even when tilted by an infinitesimal angle. This simple flow cessation phenomenon illustrates the difficulty in arriving at a macroscopic description for granular flows in a manner similar to the Navier–Stokes equations for Newtonian fluids.

For the purposes of the present review, a granular material is defined as one in which the grains interact only when they are in physical contact. Viscous and inertial forces exerted by the interstitial fluid, electrostatic, van der Waals and other forces are neglected. The contact forces between grains are usually modelled by spring-dashpot models, where the surfaces of rigid grains in contact are permitted to overlap, and the resistive force opposing overlap contains a 'spring' component proportional to the overlap distance, and a 'damping' component proportional to the relative velocity between the grains. More sophisticated models track the tangential and the normal displacements of the surfaces in contact, and resistive forces are exerted both tangential and normal to the surfaces in contact, as discussed in Section 2. The crucial difference between grain interactions and molecular interactions in a fluid is that energy is dissipated in the interaction between grains, and so a constant supply of energy is necessary to agitate the grains and sustain the flow. A further simplification is to consider the interactions between grains as instantaneous binary collisions, where energy is dissipated due to the inelastic nature of the collisions. In the instantaneous collision model, there is no intrinsic time scale associated with the period of interactions between particles. Despite this reduction in the dimensionality of the problem, many of the complex features of granular flow can be reproduced in simulations where collisions are considered instantaneous.

One of the defining ideas in the kinetic theory of sheared granular flows has its origins, ironically, in the study of dense liquid suspensions by Bagnold [8,9]. In his experimental studies on the shear and normal stresses in sheared liquid suspensions in a concentric cylinder rheometer, Bagnold identified two regimes. In the macro-viscous regime at relatively low particle concentrations, the stress was found to be proportional to the strain rate. However, at high concentrations in the 'grain-inertia' regime, Bagnold reported that the stresses are proportional to the square of the strain rate. The rationale for this non-linear dependence on the strain rate was based on a collisional argument—the frequency of collisions is proportional to the difference in velocity between two adjacent streamlines which in turn is proportional to the strain rate, while the impulse (momentum transferred per collision) is also proportional to the strain rate. Since the stress (momentum transported per unit area per unit time) is proportional to the product of the impulse and the collision frequency, there results a regime where the 'Bagnold law' is applicable, that is, the stress is proportional to the square of the strain rate.

In a subsequent experimental study of the recreation of Bagnold's experiments, Hunt et al. [10] found that the non-linear dependence of the stress on strain rate may have been an artefact of a secondary flow generated in the apparatus, due to the relatively small ratio of the height of the suspension and the width of the gap between the cylinders. Nevertheless, based on dimensional analysis, a simpler justification can be provided for the Bagnold law for dry granular flows. If we consider a collisional shear flow of a granular material in which the particles interact only through instantaneous collisions, and in the absence of other forces (viscous, electrostatic, van der Waals, etc.), there is no material time scale associated with the granular material, and the only time dimension is the inverse of the strain rate. Therefore, the Bagnold law, that the stress is proportional to the square of the strain rate, is a dimensional necessity. Though simple dimensionless analysis provides a constitutive relation between the stress and the strain rate, it is very difficult to extend this to more complex situations where the strain rate tensor has multiple components that are spatially varying. The purpose of kinetic theory is to start from a microscopic particle contact model, and derive macroscopic mass, momentum and energy conservation equations by statistical techniques. In Section 2, the instantaneous collision model and its applicability for dense granular flows is examined. The kinetic theory framework for dilute granular flows is summarised in Section 3, followed by a discussion of the extension to dense granular flows in Section 4.

2. Particle contact laws and the instantaneous collision assumption

The most commonly used model for particle-level simulations is the spring-dashpot model proposed by Cundall and Strack [11–14]. This model forms the basis of the Discrete Element Method (DEM) simulation procedure [15–18], which is commonly used for simulating dense granular flows. The particles are usually considered spherical, though more complex shapes are simulated by 'sticking' together spheres. There is an inter-particle force only when there is particle overlap, and the resistance to overlap has two components, a 'spring' component proportional to the overlap distance and a 'damping'