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Cosmic inflation / Inflation cosmique

Escher in the Sky

Escher dans le ciel

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ABSTRACT

We give a brief review of the history of inflationary theory and then concentrate on the recently discovered set of inflationary models called cosmological α-attractors. These models provide an excellent fit to the latest observational data. Their predictions $n_s \approx 1 - 2/N$ and $r \approx 12\alpha/N^2$ are very robust with respect to the modifications of the inflaton potential. An intriguing interpretation of α-attractors is based on a geometric moduli space with a boundary: a Poincaré disk model of a hyperbolic geometry with the radius $\sqrt{3\alpha}$, beautifully represented by the Escher's picture Circle Limit IV. In such models, the amplitude of the gravitational waves is proportional to the square of the radius of the Poincaré disk.

RÉSUMÉ

Après une brève revue de l'histoire de la théorie de l'inflation, cet article introduit une classe de modèles inflationaires récemment découverte : les « attracteurs de type α ». Ces modèles offrent un très bon accord avec les données observationnelles. Leur prédiction sur l'indice spectral et le rapport tenseur–scalaire, $n_s \approx 1 - 2/N$ et $r \approx 12\alpha/N^2$, est très robuste vis-à-vis d'une modification du potentiel de l'inflaton. Une interprétation surprenante de ces attracteurs α repose sur la géométrie des espaces de modules avec bord : celle d'un disque hyperbolique de Poincaré de rayon $\sqrt{3\alpha}$, merveilleusement représenté par le dessin *Circle Limit IV* d'Escher. Dans ces modèles, l'amplitude des ondes gravitationnelles est proportionnelle au carré du rayon du disque de Poincaré.

1. Introduction

During the last 35 years, inflationary theory evolved from something that could look like a beautiful science fiction story to the well established scientific paradigm describing the origin of the universe and its large scale structure. Many of its predictions have been already confirmed by observational data, see e.g. [1,2]. And yet the development of this branch of science is not over. In this paper we will briefly remember the first steps of its development, and then relate them to a

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broad set of inflationary models which seem to fit observational data particularly well, and which make predictions nearly independent on the shape of their inflationary potentials. We called these theories "cosmological attractors." As we will show, this class of models is closely related to some of the pioneering inflationary models such as the simplest versions of the chaotic inflation scenario [3,4] and the Starobinsky model [5]. But what makes these theories especially interesting is their geometric nature and supergravity realization, bringing us back to the discussion of the Poincaré disk and Escher's paintings. To put these theories into proper context, we will remind here some basic facts from the history of development of inflationary models.

2. A brief history of inflationary ideas

The development of inflationary cosmology had more than a fair share of twists and turns, and it is very different from its first implementations. It took several years until the contours of this theory became sufficiently well established.

The first model of inflationary type was proposed by Starobinsky [5]. In its original form, it was based on adding the contribution of conformal anomaly to the Einstein theory, which required existence of enormous number of different types of elementary particles contributing to the anomaly. Instead of attempting to solve the homogeneity and isotropy problems, which is the defining feature of all inflationary models, Starobinsky assumed that the universe was homogeneous and isotropic from the very beginning, and emphasized that his scenario was "the extreme opposite of Misner's initial chaos" [5]. The goal of the model was to solve the singularity problem by starting the evolution in a non-singular de Sitter state. However, dS state in his scenario was unstable, with a finite decay time [6], and therefore it could not exist at $t \to -\infty$.

The main goals of inflationary theory were formulated for the first time in the context of old and new inflation [7–9]. These models were based on an assumption that the universe initially was in a state of thermal equilibrium at an extremely high temperature, and then it supercooled and inflated in a state close to the top of the potential $V(\phi)$. At that time, this assumption seemed established beyond any reasonable doubt. However, old inflation did not quite work, as pointed out by its author [7], and it did not lead to perturbations of the cosmic microwave background radiation, which were predicted in [6,10] and discovered by COBE, WMAP and Planck. New inflation resolved most of the problems of old inflation, but it was also ruled out a year later, for many reasons discussed in [4]. After the first successes of inflationary theory, its future could appear quite bleak. As Hawking said in his book back in 1988, "the new inflationary model is now dead as a scientific theory, although a lot of people do not seem to have heard about its demise and write papers as if it were viable" [11].

The situation changed with the invention of the chaotic inflation scenario [3]. It was proposed as an alternative to new inflation, after it was realized that the assumption of the hot Big Bang, high temperature phase transitions and supercooling did not help to formulate a successful inflationary theory. In fact, these basic assumptions, the standard trademarks of old and new inflation, made inflation much more difficult to implement. If, instead, one simply considers the universe with different initial conditions in its different parts (or different universes with different values of fields in each of them), one finds that in many of them inflation may occur. It makes these parts exponentially large, thus producing exponentially large islands of order from the primordial chaos. Hence the name: chaotic inflation.

An important feature of this scenario is its versatility and the broad variety of models where it can be implemented. Examples of chaotic inflation models proposed in 1983–1985 included models with monomial and polynomial potentials, and any other models where the slow roll regime was possible. This regime is possible in small field models, with the potentials of the new inflation type, or with models with the Higgs-like potential $\sim \lambda(\phi^2 - v^2)^2$ with $v \gg 1$ [12]. Models of that type later have been called "hilltop inflation" [13]. Another example was the supergravity-based version of chaotic inflation with the potential $V \sim a(1 - \mathrm{e}^{-b\phi})$ [14], which originally appeared in the first realization of chaotic inflation in supergravity. In what follows we will call it the GL model. In 1983–1985, the Starobinsky model [5] experienced significant modifications. It was reformulated as a theory $R + aR^2$, and initial conditions for inflation in this theory were formulated along the lines of the chaotic inflation scenario [15–17]. This resolved the problem with initial conditions of the original version of this model. In the natural inflation scenario, the authors said that "our model is closest in spirit to chaotic inflation" [18]. The hybrid inflation scenario [19] was introduced as a specific version of the chaotic inflation scenario. Step by step, chaotic inflation replaced new inflation in its role of the main inflationary paradigm. Rather than describing some particular subset of inflationary models, it describes the most general approach to inflationary cosmology, which can easily incorporate ideas of quantum cosmology, eternal inflation, inflationary multiverse, and string theory landscape [20–33].

But this did not happen overnight. Chaotic inflation was so much different from old and new inflation that for a while it was psychologically difficult to accept. Even now, 30 years since the demise of old and new inflation, most of the college books on physics and astrophysics still describe inflation as exponential expansion in the false vacuum state during cosmological phase transitions with supercooling in Grand Unified Theories. That is why a significant part of the first book on inflation [4] was devoted to the discussion of new inflation versus chaotic inflation.

By now, this discussion is over, most of the existing models of inflation are based on the main principles of chaotic inflation. However this introduced a purely terminological issue: every new inflationary model belonging to the general class of chaotic inflation is introduced with its own name. That is why some authors invented a different classification of models and say, incorrectly, that chaotic inflation describes only models with monomial potentials, or only large field models, as opposite, e.g., to the hilltop inflation, natural inflation and hybrid inflation. In this paper we use the original definition of chaotic inflation following [3,4].