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# The imprint of inflation on the cosmic microwave background

## *Découvrir l'inflation dans le fond fossile micro-onde*

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ABSTRACT

The cosmic microwave background is the most precise and the most simple cosmological dataset. This makes it our most prominent window to the physics of the very early Universe. In this article I give an introduction to the physics of the cosmic microwave background and show in some detail how primordial fluctuations from inflation are imprinted in the temperature anisotropy and polarisation spectrum of the CMB. I discuss the main signatures that are suggesting an inflationary phase for the generation of initial fluctuations.



RÉSUMÉ

Le fond fossile micro-onde est l'ensemble de données cosmologiques les plus précises et les plus simples à interpréter. Ceci en fait notre fenêtre la plus directe sur la physique de l'univers primordial. Dans cet article, je présente une introduction à la physique du fond fossile micro-onde et je démontre comment les fluctuations primordiales de l'inflation se manifestent dans les anisotropies de la température et dans la polarisation du fond fossile. Je discute les principales observables qui présentent des indices importants vers une attribution des fluctuations initiales à une phase inflationnaire.



## 1. Introduction

As you can see from the contributions by most other authors to this volume, inflation is presently well established. It was originally introduced by Guth [1] to explain the flatness and the large entropy of the present Universe and to solve the horizon problem. Somewhat earlier, Starobinsky [2] had shown that a quasi de Sitter phase of expansion leads to the generation of gravitational waves from quantum fluctuations of the metric and somewhat later Mukhanov and Chibisov [3,4] found that also scalar fluctuations are inevitably generated during an inflationary phase.

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Contrary to the flatness, the homogeneity and isotropy and the large entropy of the Universe, which have been observed before they were explained and which therefore have to be regarded as 'post-dictions' of inflation,[1] the generation of perturbations was a prediction that has been verified for the first time by the COBE satellite [5] in 1992 and led to the Nobel Prize awarded to G. Smoot in 2006.

The COBE satellite observed the cosmic microwave background (CMB). The CMB is a background of thermal photons that, during the hot early phase of the Universe, were tightly coupled to baryons. As the Universe expands and cools, baryons eventually combine with electrons first to neutral helium and finally to neutral hydrogen. At an age of the Universe of about $\tau_{\rm dec} \sim 3 \times 10^5$ years and a redshift $z_{\rm dec} \simeq 1090$, the temperature drops below $T_{\rm dec} \simeq 3000$ K and there are no longer sufficiently many high energy photons around to keep the Universe ionized, most electrons are bound in neutral atoms. After that time, photons propagate freely into our antenna to be detected by COBE and other experiments. CMB experiments literally take a photo of the Universe when it was about $3 \times 10^5$ years young. This early time is not much after matter and radiation equality and since in a radiation dominated Universe fluctuations cannot grow, they are still very simply and linearly related to their value after a phase of primordial inflation. This renders the CMB a unique pristine probe of the physics of the very early Universe.

Therefore, inflation and observations of the CMB are intimately related. In this paper, I want to review this relation. For this, I introduce in the next section linear cosmological perturbations. Then I briefly indicate how quantum fluctuations are amplified during an inflationary phase and lead to classical fluctuations in the spacetime geometry. This topic is elaborated in much more detail in the contribution by A. Starobinsky. In Section 3, the heart of this paper, I explain how inflationary perturbations are imprinted in the CMB. In Section 4, I conclude.

For simplicity, I shall concentrate on a spatially flat Friedmann metric given by

$$ds^2 = a^2(t)\left(-dt^2 + \delta_{ij}dx^i dx^j\right) = -d\tau^2 + a^2(\tau)\delta_{ij}dx^i dx^j \qquad (1)$$

Here $a$ denotes the cosmic scale factor, $t$ is conformal time and $\tau$ is cosmic time. We denote the conformal Hubble parameter by $\mathcal{H}$ and the physical one by $H$,

$$\mathcal{H} = \frac{da/dt}{a}, \quad H = \frac{da/d\tau}{a} = a^{-1}\mathcal{H}$$

Latin indices run from 1 to three while greek indices run from 0 to 3, spatial vectors are indicated in boldface. Both, the speed of light and Planck's constant are set to unity, $c = \hbar = 1$. $M_{\rm p} = (8\pi G)^{-1/2}$ denotes the reduced Planck mass.

## 2. The generation of fluctuations during inflation

### *2.1. Linear cosmological perturbations*

The fluctuations in the cosmic microwave background are small. It is therefore a good strategy to compute them with linear cosmological perturbation theory. We consider a linearly perturbed Friedmann metric,

$$ds^2 = a^2(t)\left[\eta_{\mu\nu} + h_{\mu\nu}\right]dx^\mu dx^\nu \quad \text{with} \qquad (2)$$

$$h_{\mu\nu} = 2\begin{pmatrix} -\Psi & 0 \\ 0 & -\Phi\delta_{ij} + H_{ij}\end{pmatrix} \qquad (3)$$

Here $(\eta_{\mu\nu})$ is the flat Minkowski metric, $\Psi(t,\mathbf{x})$ and $\Phi(t,\mathbf{x})$ are the so called Bardeen potentials of scalar perturbations and $H_{ij}(t,\mathbf{x})$ with $H^i_i = \partial_i H^i_j = 0$ describes a gravitational wave. It can be shown that one can always bring the perturbed metric into this form, the longitudinal gauge as long as vector type (vorticity) perturbations can be neglected, see, e.g., [6]. The first-order perturbed Einstein equations relate the metric perturbations to the perturbations of the energy momentum tensor. They are given by

$$\frac{\rho a^2}{2M_P^2}\left(D_s + 3(1+w)\mathcal{H}V\right) = \Delta\Phi \qquad (4)$$

$$\frac{\rho a^2}{2M_P^2}(1+w)V = \mathcal{H}\Psi + \dot{\Phi} \qquad (5)$$

$$\frac{\rho a^2}{M_P^2}w\Pi^{(S)} = \Psi + \Phi \qquad (6)$$

[1] Actually the resolution of the horizon problem does not *explain* the homogeneity and isotropy of the Universe. It just renders is causally possible. How to generate it from an arbitrarily fluctuating initial spacetime is still an unsolved problem. Also for chaotic inflation, we require an initially homogeneous and isotropic patch of some small size.