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Phase transitions in models of human cooperation

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ABSTRACT

If only the fittest survive, why should one cooperate? Why should one sacrifice personal benefits for the common good? Recent research indicates that a comprehensive answer to such questions requires that we look beyond the individual and focus on the collective behavior that emerges as a result of the interactions among individuals, groups, and societies. Although undoubtedly driven also by culture and cognition, human cooperation is just as well an emergent, collective phenomenon in a complex system. Nonequilibrium statistical physics, in particular the collective behavior of interacting particles near phase transitions, has already been recognized as very valuable for understanding counterintuitive evolutionary outcomes. However, unlike pairwise interactions among particles that typically govern solid-state physics systems, interactions among humans often involve group interactions, and they also involve a larger number of possible states even for the most simplified description of reality. Here we briefly review research done in the realm of the public goods game, and we outline future research directions with an emphasis on merging the most recent advances in the social sciences with methods of nonequilibrium statistical physics. By having a firm theoretical grip on human cooperation, we can hope to engineer better social systems and develop more efficient policies for a sustainable and better future.

1. Introduction

The human race is remarkable in many ways. We are champions of cooperation [1]. We sacrifice personal benefits for the common good, we work together to achieve what we are unable to achieve alone, we are compassionate, and we are social. And through this cooperation, we have had astonishing evolutionary success. We have conquered our planet, and today there is an abundance of technological breakthroughs and innovations that make our lives better. At the same time, our societies are home to millions that live on the edge of existence. We deny people shelter, we deny people food, and we deny people their survival. We still need to learn how to cooperate better with one another. The problem, however, is that to cooperate more or better, or even to cooperate at all, is in many ways unnatural. Cooperation is costly, and exercising it can weigh heavily on individual wellbeing and prosperity. If only the fittest survive, why should one perform an altruistic act that is costly to perform but benefits another? Why should we care for and contribute to the public good if freeriders can enjoy the same benefits for free? Since intact cooperation forms the bedrock of our efforts for a sustainable and better future, understanding cooperative behavior in human societies has been declared as one of the grand scientific challenges of the 21st century [2].

In the past, Hamilton's kin selection theory has been applied prolifically to solve the puzzle of cooperation among simpler organisms [3], resting on the fact that by helping a close relative to reproduce still allows indirect passing of the genes to the next generation. Ants and bees, for example, are famous for giving up their own reproductive potential to support that of the queen [4]. Birds do cooperative breeding that prompts allomaternal behavior where helpers take care for the offspring of others [5]. Microorganisms also cooperate with each other by sharing resources and joining together to form biofilms [6]. But in nature cooperation is common not only between relatives. And this seems to be all the more true the more intelligent an organism is. Higher mammals, and humans in particular, are in this respect at the top of the complexity pyramid where one can distinguish a vast variety of prosocial and antisocial behavior.

Accordingly, many other mechanisms have been identified that promote cooperation, most famous being direct and indirect reciprocity as well as group selection [7]. Network reciprocity [8] has recently also attracted considerable attention in the physics community, as it became clear that methods of nonequilibrium statistical physics can inform relevantly on the outcome of evolutionary

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games in structured populations [9–13]. While the basic idea behind network reciprocity is simple — cooperators do better if they are surrounded by other cooperators — the manifestation of this fact and the phase transitions leading to it depend sensitively on the structure of the interaction network and the type of interactions, as well as on the number and type of competing strategies.

While the infusion of statistical physics to this avenue of research is still a relatively recent development, evolutionary game theory [14] is long established as the theory of choice for studying the evolution of cooperation among selfish individuals, including humans [15]. Competing strategies vie for survival and reproduction through the maximization of their utilities, which are traditionally assumed to be payoffs that are determined by the definition of the contested game. The most common assumption underlying the evolution in structured populations has been that the more successful strategies are imitated and thus spread based on their success in accruing the highest payoffs. Mutation has also been considered prominently, in that it can reintroduce variation into the population or represent cultural evolution and social learning, in which people imitate those with higher payoffs and sometimes experiment with new strategies. Evolutionary dynamics based on these basic principles has been considered as the main driving force of evolution, reflecting the individual struggle for success and the pressure of natural selection.

Undoubtedly, traditional evolutionary game theory, as briefly outlined above, has provided fundamental models and methods that enable us to study the evolution of cooperation, and research along these lines continues to provide important proof-of-principle models that guide and inspire future research. But the complexity of such systems also requires methods of nonequilibrium statistical physics be used to better understand cooperation in human societies, and to reveal the many hidden mechanisms that promote it. In the continuation, we first present the public goods game on the square lattice as the null model of human cooperation [16]. We then proceed with representative extensions of the game involving punishment [17] and correlated positive and negative reciprocity [18], which deliver fascinating examples of phase transitions in the realm of this research. We conclude with an overview of important progress made in related fields, and we outline possible directions for future research in the realm of statistical physics of evolutionary games.

2. The null model

The public goods game is simple and intuitive. In a group of players, each one can decide whether to cooperate or defect. Cooperators contribute $c = 1$ to the common pool, while defectors contribute nothing. The sum of all contributions is multiplied by a multiplication factor $r > 1$, which takes into account synergistic effects of cooperation. In particular, there is an added value to a joint effort that is often more than just the sum of individual contributions. After the multiplication, the resulting amount of public goods is divided equally amongst all group member, irrespective of their strategy. In a group g containing G players the resulting payoffs are thus

$$\Pi_C^g = r(N_C + 1)/G - 1 \qquad (1)$$

$$\Pi_D^g = rN_C/G, \qquad (2)$$

where N_C is the number of cooperators around the player for which the payoff is calculated. Evidently, the payoff of a defector is always larger than the payoff of a cooperator, if only $r < G$. With a single parameter, the public goods game hence captures the essence of a social dilemma in that defection yields highest short-term individual payoffs, while cooperation is optimal for the group, and in fact for the society as a whole. If nobody cooperates public goods vanish and we have the tragedy of the commons [19].

In a well-mixed population, where groups are formed by selecting players uniformly at random, $r = G$ is a threshold that marks the transition between defection and cooperation. If players imitate strategies of their neighbors with a higher payoff, then for $r < G$ everybody defects, while for $r > G$ everybody in the population cooperates. Interactions among humans, however, are seldom random, and it is therefore important for the null model to take this into account. The square lattice is among the simplest of networks that one can consider. Notably, previous research has shown that for games governed by group interactions using the square lattice suffices to reveal all feasible evolutionary outcomes, and moreover, that these are qualitatively independent of the details of the interaction structure [11].

For simplicity but without loss of generality, let the public goods game thus be staged on a square lattice with periodic boundary conditions where L^2 players are arranged into overlapping groups of size $G = 5$ such that everyone is connected to its $G - 1$ nearest neighbors. The microscopic dynamics involves the following elementary steps. First, a randomly selected player x with strategy s_x plays the public goods game with its $G - 1$ partners as a member of all the $g = 1, \ldots, G$ groups where it is member, whereby its overall payoff Π_{s_x} is thus the sum of all the payoffs $\Pi_{s_x}^g$ acquired in each individual group. Next, player x chooses one of its nearest neighbors at random, and the chosen co-player y also acquires its payoff Π_{s_y} in the same way. Finally, player y imitates the strategy of player x with a probability given by the Fermi function

$$W(s_x \to s_y) = \frac{1}{1 + \exp[(\Pi_{s_y} - \Pi_{s_x})/K]}, \qquad (3)$$

where K quantifies the uncertainty by strategy adoptions [16]. In the $K \to 0$ limit, player y copies the strategy of player x if and only if $\Pi_{s_x} > \Pi_{s_y}$. Conversely, in the $K \to \infty$ limit, payoffs cease to matter and strategies change as per flip of a coin. Between these two extremes players with a higher payoff will be readily imitated, although under-performing strategies may also be adopted, for example due to errors in the decision making, imperfect information, and external influences that may adversely affect the evaluation of an opponent. Repeating these elementary steps L^2 times constitutes one full Monte Carlo step (MCS), which gives a chance to every player to change its strategy once on average.

This null model — the spatial public goods game — has been studied in detail in [16], where it was shown that for $K = 0.5$ cooperators survive only if $r > 3.74$, and they are able to defeat defectors completely for $r > 5.49$. Both the $D \to C + D$ and the $C + D \to D$ phase transition are continuous. Subsequently, the impact of critical mass [20], i.e., the evolution of cooperation under the assumption that the collective benefits of group membership can only be harvested if the fraction of cooperators within the group exceeds a threshold value, and the effects of different group sizes [21], have also been studied in the realm of this two-strategy spatial public goods game.

In general, it is important that in structured populations, due to network reciprocity, cooperators are able to survive at multiplication factors that are well below the $r = G$ limit that applies to well-mixed populations. The $r > 3.74$ threshold for cooperators to survive on the square lattice can be considered as a benchmark value, below and above which we have harsh and lenient conditions for the evolution of public cooperation, respectively.

3. Public goods game with punishment

Despite ample cooperation in human societies [1], and despite our favorable predispositions for prosocial behavior that are likely