Physics Letters A 380 (2016) 2861–2869

Contents lists available at ScienceDirect

Physics Letters A

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Enhancing Casimir repulsion via topological insulator multilayers

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ARTICLE INFO

Article history:
Received 6 December 2015
Received in revised form 1 June 2016
Accepted 2 June 2016
Available online 6 June 2016
Communicated by V.A. Markel

Keywords:
Repulsive Casimir force
Topological insulator
Topological magnetoelectric effect
Multilayer structure

ABSTRACT

We propose to observe the enhanced Casimir repulsion between two parallel multilayer walls made of alternating layers of a topological insulator (TI) and a normal insulator. Based on the transfer matrix method, the Fresnel coefficients matrix is generalized to apply to the TI multilayer structure. The Casimir repulsion under the influence of the magnetization orientation in the magnetic coatings on TI layer surfaces, the layer thicknesses, and the topological magnetoelectric polarizability, is investigated. We show that, for the multilayer structures with parallel magnetization on the TI layer surfaces, it is possible to enhance the repulsion by increasing the TI layer number, which is due to the accumulation of the contribution to the repulsion from the polarization rotation effect occurring on each TI layer surface. Generally, in the distance region where there is Casimir attraction between semi-infinite TIs, the force may turn into repulsion in TI multilayer structure, and in the region of repulsion for semi-infinite TI, the repulsive force can be enhanced in magnitude, the enhancement tends to a maximum while the structure contains sufficiently many layers.

1. Introduction

With the recent advances in microelectromechanical and nanoelectromechanical systems (MEMS and NEMS), the fluctuation-induced Casimir force [1] that becomes relevant between neutral macroscopic objects at micron and submicron scales has drawn increased attention. Progress has been made in the theoretical understanding and experimental measurements of the Casimir effect for various materials and geometries [2,3]. The Casimir force provides possible applications for nanotechnology; however, the stiction problem may also arise in the MEMS and NEMS due to the well-known Casimir attraction [4–6]. A variety of ideas have been proposed to change the sign of the force, among which an important method works by setting the special electromagnetic responses of the media. A famous example is that a perfectly conducting plate repels an infinitely permeable plate in vacuum [7]. For practical situations, the Casimir repulsion is possible to be obtained by using magnetodielectric media or metamaterials [8–11]. The repulsion between two plates immersed in a dielectric fluid (first proposed in Ref. [12]), instead of involving magnetic material, has been measured experimentally [13].

Recently, a new quantum state of matter named topological insulator (TI) is adopted to obtain the repulsive Casimir force [14–19]. Three-dimensional TI has a full gap in the bulk like normal insulator (NI), and has gapless surface states with odd Dirac cones protected topologically by the time-reversal symmetry [20–25], which has been discovered to exist in thermoelectric materials, e.g., Bi_2Te_3, Sb_2Te_3, and $TlBiSe_2$ compounds [22,24–27]. The concept of TI can be defined in the topological field theory that describes a quantized magnetoelectric response, known as the topological magnetoelectric effect [28]. Within the theory, the Maxwell Lagrangian of classical electromagnetism is modified by a topological term $\alpha\Theta\mathbf{E}\cdot\mathbf{B}/(4\pi^2)$, where α is the fine structure constant, and $\Theta = (2n+1)\pi$ is the topological magnetoelectric polarizability (TMEP), n is an integer. To adapt such definition and to obtain the Casimir repulsion, a thin magnetic coating needs to be added on the TI surface to open the band gap, and the direction of magnetization determines the sign of Θ. It has been discovered that the Casimir repulsion is greatly increased in the case of the larger value of TMEP [14,15]; but how to obtain such material with large Θ is still an open question. Based on the effective model, one can arbitrarily realize various different topologically nontrivial phases, simply by changing its model parameters. However, it is practically not that simple because different model parameters correspond to different material. Motivated by these problems, in this work, we propose and examine a Casimir system considering TI multilayer structure, and the effect of our proposal is, in some

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http://dx.doi.org/10.1016/j.physleta.2016.06.002
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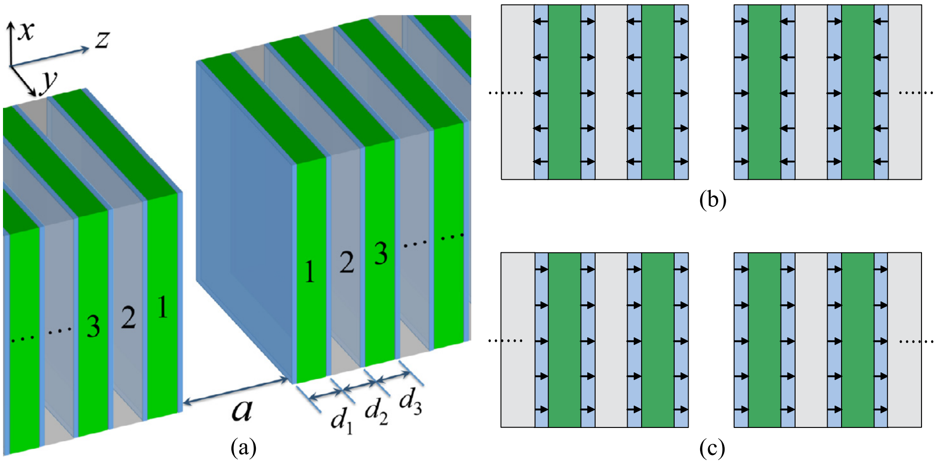

(a)

(b)

(c)

Fig. 1. Schematic of the Casimir system considered: (a) Two TI (green)–NI (gray) multilayer structures separated by a distance a. Every TI layer is covered with thin magnetic coating (blue), the thickness of which is much smaller than the separation a and the thicknesses of TI and NI. The directions of magnetization can be either (b) antiparallel or (c) parallel. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

sense, equivalent to increasing the Θ value. The bulk state of TI has no difference from the normal insulator, and consequently offers little contribution to the Casimir repulsion. The repulsive force is attributed to the TI surface effect. The TI multilayer structure can effectively increase the surface share of the total system, therefore the Casimir repulsion may be easily obtained. The TI multilayer is also recently investigated in the realization of a three-dimensional Weyl fermion [29], and in engineering a system to realize robust weak TI phase [30]. Here through the investigation of the Casimir–Lifshitz interaction between the multilayered TIs, we show that in the case of TI layer with parallel magnetization on its two surfaces, the TI multilayer structure has an enhancing effect on the Casimir repulsion. In the distance region of repulsion for semi-infinite TI, the repulsive force can be enhanced in magnitude for multilayer structure, the enhancement tends to a maximum while the structure contains sufficiently many layers (85 TI layers for $|\Theta| = \pi$, see Section 3): For the realistic TI, for example, TlBiSe$_2$ [26], at a distance of 200 nm, the maximum achieved repulsion is $\sim -1 \times 10^{-5}$ N·m^{-2}, while for the semi-infinite TI the force is $\sim -7 \times 10^{-6}$ N·m^{-2}. Although at that distance it may not be a very remarkable enhancement, the force in the distance region where there will be Casimir attraction between semi-infinite TIs may turn into repulsion for the TI multilayer structure. We also study how the layer thickness and the value of Θ influence this multilayer effect.

2. Transfer matrix formalism for topological insulator multilayer

In what follows, a Casimir system composed of two TI–NI multilayer structures, separated by a vacuum region of width a, is considered as shown in Fig. 1. Thin magnetic coating is covered on every TI surface only to open the surface band gap without inducing any other effect. The NI layers separate every two TI layers in order that the magnetic coating covering the TI layer surface does not affect the other adjacent TI layers. The magnetization direction of the coating may be either parallel or antiparallel to the z direction, and both of the cases will be studied below. The Casimir force per unit area between two multilayers can be expressed in terms of the integral of imaginary frequency ξ ($\omega = i\xi$) [31]:

$$\frac{F_C(a)}{A\hbar} = \int_0^\infty d\xi \int \frac{d^2\mathbf{k}_\parallel}{4\pi^3} K_0 \mathrm{Tr}\frac{\mathbf{R}_l \cdot \mathbf{R}_r e^{-2K_0 a}}{1 - \mathbf{R}_l \cdot \mathbf{R}_r e^{-2K_0 a}}, \tag{1}$$

where A is the surface area, $\mathbf{k}_\parallel$ is the component of the wave vector parallel to the structure surface, and $K_0 = \sqrt{k_\parallel^2 + \xi^2/c^2}$. $\mathbf{R}_{l(r)}$ is 2×2 Fresnel coefficients matrix

$$\mathbf{R}_{l(r)} = \begin{pmatrix} r^{l(r)}_{\mathrm{TE,TE}} & r^{l(r)}_{\mathrm{TE,TM}} \\ r^{l(r)}_{\mathrm{TM,TE}} & r^{l(r)}_{\mathrm{TM,TM}} \end{pmatrix}, \tag{2}$$

where $r^{l(r)}_{u,v}$ ($u, v = \mathrm{TE}, \mathrm{TM}$) is the reflection amplitude of an incident u mode which is reflected in a v mode by the left (right) TI multilayer. To find the matrices $\mathbf{R}$, here a transfer matrix formalism is adopted to analyze the electromagnetic wave propagations through the layered TI system: Based on the Maxwell equations with surface corrections for the electromagnetic field in the presence of the TIs, a wave function vector can be written to express the electromagnetic waves. Since in the structure containing TI the nonzero reflection and transmission coefficients showing polarization-mixing are induced, thus the wave function vector of frequency ω should be four-component, which in the jth layer is defined as $\Psi_j(\mathbf{r}, \omega) = \left[E_{xj}(\mathbf{r}, \omega), E_{yj}(\mathbf{r}, \omega), H_{xj}(\mathbf{r}, \omega), H_{yj}(\mathbf{r}, \omega)\right]^T$ with $E_{x(y)j}$ and $H_{x(y)j}$ the x (y) components of the electric and magnetic fields, respectively.

Then within the framework of the transfer matrix method, the propagation matrix that describes the wave function vector $\Psi(\mathbf{r}, \omega)$ propagating over a distance in a homogeneous medium, and the transmission matrix that connects the vectors across an interface, are determined by imposing the modified Maxwell equations and boundary conditions with the topological terms. Eventually by considering the connection between $\Psi(\mathbf{r}, \omega)$ in the incident/reflected field and that in the transmitted field, the Fresnel coefficients can be expressed in terms of the elements of the propagation and transmission matrices. It is noted that in the situation of TI layer with parallel magnetization on its two surfaces, there are domain walls on the edge of the layer slab where a quantum Hall edge current is induced [28], and thus the contribution of the current should be included in the Maxwell equations. However, given the fact that the domain wall exists at the far edge of the TI layer, the influence of the edge current can then be neglected here in the investigation for the Casimir system of the infinitely extended planar structure without considering the edge effect.

After the above derivation (details are given in Appendix A), one can get the propagation matrix of the TI or NI jth layer