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Origin and dynamics of a bottleneck-induced shock in a two-channel exclusion process

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ABSTRACT

We analyze the origin and characteristics of the bottleneck-induced shock in a two-channel totally asymmetric simple exclusion process with Langmuir kinetics under symmetric coupling conditions. The variation in height of the spike, which has been found to be a precursor to the bottleneck-induced shock, is analyzed with respect to lane-changing rate Ω and bottleneck rate q. The critical value of $q(q_c)$, below which the effect of bottleneck turns from local to global, has been identified. A non-monotonic variation of q_c with respect to Ω is observed. The bottleneck-induced shock exhibits turning effect with respect to an increase in Ω.

1. Introduction

Totally asymmetric simple exclusion process (TASEP) is the simplest model for studying out-of-equilibrium processes ranging from traffic flow [1] to biological transport [2], in which particles hop in a preferred direction along the lattice. The usefulness of this approach can be seen from its suitability to describe successfully various physical, chemical and biological processes such as kinetics of bio-polymerization [2], protein synthesis [3], dynamics of motor proteins [4], diffusion through membrane [5], gel electrophoresis [6], vehicular traffic [7] and modeling of ant-trails [1] etc. Despite their simplicity, these models are competent to efficiently explain some complex non-equilibrium phenomena such as boundary-induced phase transitions [8], phase separation [9], spontaneous symmetry breaking [10] and localized shocks [11–13] etc.

The boundary-induced phase transition, which is the concern of present study, is the appearance of a shock in exclusion process models. A shock or a domain wall is a discontinuity over a microscopic regime in the bulk connecting a low-density state to a high-density state. The domain wall theory [14–16] gives a phenomenological description of the shock dynamics and allows us to understand the steady-state dynamics of driven diffusive systems. In a single-channel TASEP, system exists in three stationary phases low-density (LD), high-density (HD) and maximal current (MC). The LD and HD phases are separated by a LD/HD phase coexistence line. The domain wall connecting the two states is delocalized, since it explores the whole system diffusively [14]. However, in a single-channel TASEP with Langmuir kinetics (LK) [11,17], one finds localized shocks in the steady-state. Instead of a coexistence line in the phase diagram of TASEP without LK, as discussed above [14], there exists a coexistence region in the phase-plane for TASEP with LK.

Shocks can also occur in multi-channel TASEPs with as well as without LK [18–23,13]. The interesting feature of a two-channel TASEP without LK is the emergence of a localized shock, which distinguishes it from its single-channel counterpart. However, the above statement is true under a specific coupling environment. In particular, under symmetric lane-changing rule, the two-channel TASEP reduces to two independent single-channel TASEPs, which rules out the possibility of existence of shock. Interestingly, there exists a localized shock under weakly asymmetric coupling conditions [13], which vanishes in the strong asymmetric coupling limit [13,24–26]. However, in the presence of Langmuir kinetics, there exist localized shock in two-channel TASEP under all coupling conditions [18,19]. The above discussed results are summarized in the form of Table 1, which indicates a clear idea about the existence of shock in a TASEP under different conditions of coupling as well as Langmuir kinetics.

The literature reported above has focused on a homogeneous TASEP, where the rate of forward hopping of particles in the bulk is uniform throughout the lattice. Moving our attention to the central theme of the current paper, we outline the importance of studying inhomogeneous transport systems as follows. In protein synthesis,

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Table 1
Existence of a shock in ASEPs under different coupling conditions. Here, ✗(✓) denotes absence (presence) of a localized shock.

Number of lanes	Type of coupling	Without LK	With LK
1	No coupling	✗ [14]	✓ [17]
2	Symmetric	✗ [27]	✓ [28]
	Strong asymmetric	✗ [13,24–26]	✓ [18]
	Weakly asymmetric	✓ [13]	✓ [19]

the genetic information is deciphered into proteins by molecular machines called ribosomes, which attach themselves at the start end of mRNA, move along the chain in a unidirectional manner and finally detach at the stop end [5]. The important factor affecting the ribosome translation rate is relative concentrations of transfer-RNA (tRNA), which may vary from codon to codon. The codons with lower concentrations of tRNA are called slow codons, which inhibit the protein synthesis playing the role of an inhomogeneity in a homogeneous system [29–31]. Apart from the context of protein synthesis, inhomogeneities also occur naturally in many other transport systems such as vehicular traffic and flow of data in a network. In traffic flow, the ongoing construction on roads, a slow moving vehicle or an accident can lead to slow down the flow rate on highways and can lead to congestion. Similarly, the presence of a bottleneck link in a network limits the underlying capacity of the path from client to server causing network congestion [32].

Several studies have been performed on single-channel inhomogeneous TASEPs with [33,34] as well as without LK [3,5,35–41]. While investigating the role of a bottleneck in a closed TASEP without LK [37,38], it is found that even the presence of a single bottleneck site can produce the shock profile and a plateau in the fundamental current-density relation. The study of a single-channel open inhomogeneous ASEP without LK has revealed the existence of a HD/LD coexistence phase [39,41]. A detailed study to analyze the effect of bottleneck in an open single-channel TASEP with LK has been carried out by Pierobon et al. [33], in which the emergence of a bottleneck-induced shock is reported. Owing to its name, it is clear that this shock is not produced due to deconfinement of any boundary layer but is generated in the system due to the bottleneck. Later, the role of bottleneck in a two-channel symmetrically coupled TASEP with LK has been analyzed by Wang et al. [42]. It is observed that the transition rate through bottleneck as well as lane-changing rate affects the steady-state dynamics significantly. Recently [43], two-channel TASEP with LK has been investigated thoroughly with the introduction of a new hybrid mean-field approach. The steady-state phase diagrams and density profiles have been derived and the effect of various kinetic rates has also been analyzed. Additionally, the existence of a bottleneck-induced shock in both the lanes has been reported.

The objective of present study is two-fold. One is to discuss the formation of bottleneck-induced shock from its origination along with mathematical quantification and secondly to examine the dynamics of shock with respect to bottleneck rate and lane-changing rate. The analysis also attempts to unveil the differences or similarities of the steady-state behavior of bottleneck-induced shock with the one in corresponding single-channel system.

2. Model description

The model is defined as a two-channel $(L, 2)$ lattice, where L is the length of each channel. Adopting random-sequential update rules, a lattice site (i, j); $i = 1, 2, 3, \ldots.. L$; $j = A, B$ is randomly chosen at each time step. The state of the system is characterized by a set of occupation numbers $\tau_{i,j}$ $(i = 1, 2, 3, \ldots.. L;\ j = A, B)$, each of which is either zero (vacant site) or one (occupied site). At entrance $(i = 1)$, a particle can enter the lattice with a rate α

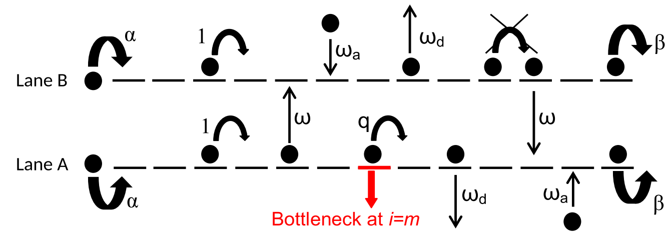

Fig. 1. Schematic diagram of a symmetrically coupled two-channel TASEP with LK with bottleneck in lane A at mth site. Crossed arrows show forbidden transitions.

provided $\tau_{1,j} = 0$; and at exit $(i = L)$, a particle can leave the lattice with a rate β when $\tau_{L,j} = 1$. In the bulk, if $\tau_{i,j} = 1$, then the particle at the site (i, j) firstly tries to detach itself from the system with a rate ω_d (detachment rate) and if it fails then it moves forward to site $(i+1, j)$ with a rate $p_{i,j}$ provided $\tau_{i+1,j} = 0$; otherwise it attempts to shift to other lane with a rate ω, only if the target site is vacant. On the other hand, if $\tau_{i,j} = 0$; $i = 2, 3, \ldots, L-1$, a particle can attach to the site (i, j) with a rate ω_a (attachment rate). Here, horizontal transition rate $p_{i,j}$ is inhomogeneous and is given by the following binary distribution

$$p_{i,j} = \begin{cases} q\,; & i = m \ \&\ j = A \\ 1\,; & \text{otherwise} \end{cases} \tag{1}$$

The bottleneck connects the sites at $i = m$ and $i = m + 1$ in lane A (Fig. 1), while lane B is kept homogeneous. Here, q (bottleneck rate) denotes the transition rate of a particle on passing through the bottleneck, $\omega_{a,d}$ denotes attachment and detachment rate and ω denotes the lane-changing rate. Note that the same model as defined in Ref. [43] has been undertaken here for analysis and discussed for the self-containment of the paper.

3. Steady-state mean-field equations

To observe the competing interplay between attachment–detachment and particle injection and removal dynamics, we rescale the time and other kinetic rates as $t' = t/L$, $\Omega_a = \omega_a L$, $\Omega_d = \omega_d L$ and $\Omega = \omega L$ [18,17]. The derivation of the following steady-state hybrid system of equations, which has been formulated in Ref. [43], is skipped here to avoid repetition.

$$\begin{aligned}
&\frac{\epsilon}{2}\frac{d^2\rho_A}{d\hat{x}^2} + (2\rho_A - 1)\frac{d\rho_A}{d\hat{x}} + \Omega_d(K - (K+1)\rho_A) \\
&\quad - \Omega\rho_A^2(1-\rho_B) + \Omega\rho_B^2(1-\rho_A) = 0, \\
&\rho_{m-1,A}(1-\rho_{m,A}) - q\rho_{m,A}(1-\rho_{m+1,A}) \\
&\quad + \omega_d(K - (K+1)\rho_{m,A}) - \omega\rho_{m,A}\rho_{m+1,A}(1-\rho_{m,B}) \\
&\quad + \omega\rho_{m,B}\rho_{m+1,B}(1-\rho_{m,A}) = 0, \\
&q\rho_{m,A}(1-\rho_{m+1,A}) - q\rho_{m+1,A}(1-\rho_{m+2,A}) \\
&\quad + \omega_d(K - (K+1)\rho_{m+1,A}) - \omega\rho_{m+1,A}\rho_{m+2,A}(1-\rho_{m+1,B}) \\
&\quad + \omega\rho_{m+1,B}\rho_{m+2,B}(1-\rho_{m+1,A}) = 0, \\
&\frac{\epsilon}{2}\frac{d^2\rho_B}{dx^2} + (2\rho_B - 1)\frac{d\rho_B}{dx} + \Omega_d(K - (K+1)\rho_B) \\
&\quad + \Omega\rho_A^2(1-\rho_B) - \Omega\rho_B^2(1-\rho_A) = 0,
\end{aligned} \tag{2}$$

where $\epsilon = 1/L$ is lattice constant, $K = \Omega_a/\Omega_d$ is the binding constant, $0 < \hat{x} < m\epsilon$, $(m+1)\epsilon < \hat{x} < 1$ and $0 < x < 1$. For a detailed solution methodology, we again refer the readers to read Ref. [43].