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On the macroscopic quantization in mesoscopic rings and single-electron devices

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ABSTRACT

In this letter we investigate the phenomenon of macroscopic quantization and consider particle on the ring interacting with the dissipative bath as an example. We demonstrate that even in presence of environment, there is macroscopically quantized observable which can take only integer values in the zero temperature limit. This fact follows from the total angular momentum conservation combined with momentum quantization for bare particle on the ring. The nontrivial thing is that the model under consideration, including the notion of quantized observable, can be mapped onto the Ambegaokar–Eckern–Schon model of the single-electron box (SEB). We evaluate SEB observable, originating after mapping, and reveal new physics, which follows from the macroscopic quantization phenomenon and the existence of additional conservation law. Some generalizations of the obtained results are also presented.

1. Introduction

The phenomenon of Coulomb blockade [1–6] in different meso- and nanostructures is one of the most striking manifestations of the charge quantization. This phenomenon can be observed, for example, in the so called single-electron box (SEB). This system consists of small metallic grain, which is coupled to lead by tunnel junction and capacitively to gate electrode. One can change the number of electrons on the island by tuning the gate voltage. However, the charge of the island can be changed only by tunneling of one additional electron from the lead. It means that the total charge of the grain is equal to an integer number of elementary electron charges e, or, in other words, charge is quantized. This phenomenon can be observed if the temperature of the system is much lower than the Coulomb energy of the grain. The simplest model Hamiltonian, which describes this system, is given by Coulomb energy operator

$$\hat{H} = E_C(\hat{N} - CV_g/e)^2, \tag{1}$$

where C is a capacitance of the island, $E_C = e^2/(2C)$ is a charging energy, V_g is a gate voltage. Here we neglect the change of electron energy due to tunneling. This can be justified if mean level spacing of the island is the smallest energy scale of the system. $\hat{N}$ is an operator of the number of excess electrons in the grain. Its spectrum is represented by integer numbers, so the charge of the grain $\hat{q} = e\hat{N}$ is quantized. Let us note that this Hamiltonian becomes similar to the Hamiltonian of the charged particle on the ring if one makes the substitution $\hat{N} \to \hat{p}$, $E_C \to 1/(2mR^2)$, where m is the particle mass, R is a ring radius and $\hat{p}$ is an operator of the particle angular momentum, which is canonically conjugated to position operator $\hat{\theta}$

$$[\hat{p}, \hat{\theta}] = -i. \tag{2}$$

In coordinate representation θ varies from 0 to 2π and $\hat{p} = -i\partial/\partial\theta$. The last object, which should be identified in order to complete the mapping is the combination $q_x = CV_g/e$. It is precisely the magnetic flux piercing the ring $q_x \to \phi_x = \Phi/\Phi_0$ in units of flux quanta $\Phi_0 = 2\pi/e$. In other words, one can observe that the SEB can be mapped onto one-channel ring with single electron in the magnetic field. The important thing is that this mapping goes far beyond the simple approximation described above and is valid even if the electron tunneling between lead and grain is properly taken into account. This case might be considered in the framework of well-known Ambegaokar–Eckern–Schon (AES) approach [7,8,1] if one employs smallness of tunnel barrier transparency. After integration over fermionic degrees of freedom one comes to the effective theory for charge degree of freedom. The resulting effective action (see below) contains in addition to the charging part, the non-local in time contribution which describes electron tunneling. It turns out that the effective action which describes particle on the ring interacting with the dissipative environment has precisely

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the same form [9–13]. Moreover, from the form of the action it follows that the environment is of Caldeira–Leggett (CL) type [14,1]. Note, that the equivalence of this two models has been extensively used by various authors in the numerical computations of the SEB properties (see for example [15,11] and references therein).

As we mentioned above the average charge on the island is quantized at zero temperature, but, strictly speaking, this statement is correct only in the limit of vanishing tunnel coupling between the lead and grain. In general case the average charge varies continuously with gate voltage changing. Recently it was argued by Burmistrov and Pruisken [16,17] (see also Refs. [18–20]) that in the SEB there is another quantity – an "effective charge" which is nevertheless quantized in the zero temperature limit. They related this quantity to the sensitivity of the system to the changing of boundary conditions. Also the unifying scaling diagram of the problem was introduced by usage of the similarity between AES theory and the theory of quantum Hall effect. Burmistrov and Pruisken supported their statements by explicit calculations in the cases of the small and large couplings.

The aim of the present letter is to prove rigorously the existence of the "effective charge"-like quantity, its quantization and to shed light on the underlying physics especially in the context of the mapping discussed above.

The paper organized as follows. At the beginning, we introduce the partition function for the particle on the ring interacting with the environment and its relation to the SEB. After that we rewrite it in the operator formalism and introduce fictitious many-particle system. On the next step we demonstrate, that the considered system has additional integral of motion and relate this conserving quantity to macroscopic quantization phenomena. At the end of the paper we generalize all obtained results to the more complicated cases and discuss their physical meaning.

2. Model and basic definitions

Let us consider particle on the ring interacting with linear dissipative environment at temperature T. It is well known that the partition function in this case can be represented through the path integral as [9–11]

$$\mathcal{Z}=\int_0^{2\pi} d\theta_0 \sum_{n=-\infty}^{\infty} e^{2\pi i n\phi_x} \int_{\theta(0)=\theta_0}^{\theta(\beta)=\theta_0+2\pi n} \mathcal{D}\theta e^{-S[\theta(\tau)]}, \tag{3}$$

where

$$S[\theta]\equiv S_0[\theta]+S_i[\theta]=\frac{mR^2}{2}\int_0^\beta \dot\theta^2(\tau)d\tau + \frac{g}{4}\int_0^\beta d\tau\int_0^\beta d\tau' \alpha(\tau-\tau')e^{i\theta(\tau)-i\theta(\tau')}. \tag{4}$$

Here m and R are the mass of the particle and the ring radius, g is the coupling constant between particle and environment, $\beta=1/T$ is the inverse temperature, and the kernel $\alpha(\tau-\tau')$ is governed by statistical and dynamical properties of the environment. Integration is performed over trajectories with a given winding number n, which are periodic up to constant $\theta(\beta)=\theta(0)+2\pi n$. $\phi_x=\Phi/\Phi_0$ is the flux piercing the ring in the units of flux quanta $\Phi_0=2\pi/e$ (here and below we set the Planck's constant and the speed of light equal to unity $\hbar=1,\ c=1$). Due to bosonic nature of the environment, kernel $\alpha(\tau)$ is the periodic function of imaginary time τ with period equal to β. It can be represented in the form

$$\alpha(\tau)=\frac{T}{\pi}\sum_{n=-\infty}^{\infty}F(\omega_n)e^{-i\omega_n\tau}, \tag{5}$$

where $\omega_n=2\pi nT$ is Matsubara frequency and function $F(z)$ is symmetric $F(z)=F(-z)$ and equals to zero at zero frequency $F(0)=0$. Usually, the last condition might be achieved by subtraction of some unimportant constant from the initial action. The CL environment [1,14] corresponds to $F(z)=|z|$. This case is very important from the practical point of view, since it is equivalent to the AES model describing the single-electron box. As we mentioned in the Introduction, in order to relate these two problems one should identify $1/(2mR^2)$ with charging energy E_C of the island, g with dimensionless conductance g_t of the tunnel junction between island and reservoir, and flux ϕ_x with the external charge q_x induced by gate voltage. Below we will consider essentially particle on the ring, but on every step we will have SEB in mind.

3. Decoupling of the action

The key idea of our approach consists in proper decoupling of the interaction term with help of fictitious many-body system interacting with the particle on the ring with Hamiltonian.

$$\hat H_f=\frac{(\hat p-\phi_x)^2}{2mR^2}+\sum_k \varepsilon_k\left(\hat a_k^\dagger\hat a_k+\hat b_k^\dagger\hat b_k\right) + \sqrt{g}e^{i\hat\theta}\sum_k(\hat a_k+\hat b_k^\dagger)+\sqrt{g}e^{-i\hat\theta}\sum_k(\hat a_k^\dagger+\hat b_k). \tag{6}$$

Here $\hat\theta$ is the angle variable corresponded to the position of the particle on the ring, and $\hat p$ is the angular momentum operator. The first part of Hamiltonian describes free particle as we discussed in the Introduction. Other terms are fictitious system Hamiltonian and interaction between particle and fictitious system. It consists of two sets of bosonic degrees of freedom, which creation and annihilation operators are denoted by $\hat a_m^\dagger,\hat b_m^\dagger$ and $\hat a_m,\hat b_m$ correspondingly. Commutation relations are standard

$$[\hat a_k,\hat a_l]=[\hat a_k,\hat b_l]=[\hat b_k,\hat b_l]=[\hat a_k,\hat b_l^\dagger]=0, \tag{7}$$

$$[\hat a_k,\hat a_l^\dagger]=[\hat b_k,\hat b_l^\dagger]=\delta_{k,l}, \tag{8}$$

where $\delta_{k,l}$ is the Kroenecker symbol. In order to establish the connection between system with Hamiltonian (6) and initial system with action (4) one should make the special choice of energies ε_k. After transformation of partition function $\mathcal{Z}=\mathrm{tr}\,e^{-\beta\hat H_f}$ into path integral representation one can obtain

$$\alpha(\tau)=-4\sum_k\langle\mathcal{T}(\hat a_k(\tau)+\hat b_k^\dagger(\tau))(\hat a_k^\dagger(0)+\hat b_k(0))\rangle_{a,b}, \tag{9}$$

where averaging is performed with equilibrium non-interacting density matrix $\langle\ldots\rangle_{a,b}\equiv\mathrm{tr}(\ldots e^{-\beta\hat H_b})/\mathrm{tr}(e^{-\beta\hat H_b})$, where

$$\hat H_b=\sum_k\varepsilon_k\left(\hat a_k^\dagger\hat a_k+\hat b_k^\dagger\hat b_k\right), \tag{10}$$

$\mathcal{T}$ is the time ordering symbol, and

$$\hat a_k(\tau)=e^{\tau\hat H_b}\hat a_k e^{-\tau\hat H_b},\qquad \hat a_k^\dagger(\tau)=e^{\tau\hat H_b}\hat a_k^\dagger e^{-\tau\hat H_b}, \tag{11}$$

$$\hat b_k(\tau)=e^{\tau\hat H_b}\hat b_k e^{-\tau\hat H_b},\qquad \hat b_k^\dagger(\tau)=e^{\tau\hat H_b}\hat b_k^\dagger e^{-\tau\hat H_b} \tag{12}$$

are operators in Matsubara representation. After averaging one has

$$F(\omega_n)=-8\pi\sum_k\frac{\varepsilon_k}{\omega_n^2+\varepsilon_k^2}+C = -8\pi\int_0^\infty\frac{dz}{2\pi}\frac{zJ(z)}{\omega_n^2+z^2}+C, \tag{13}$$