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Statistical investigation and thermal properties for a 1-D impact system with dissipation

Gabriel Díaz I. *, André L.P. Livorati, Edson D. Leonel

Departamento de Física, UNESP – Univ. Estadual Paulista, Av. 24A, 1515, Bela Vista, 13506-900, Rio Claro, SP, Brazil

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ABSTRACT

The behavior of the average velocity, its deviation and average squared velocity are characterized using three techniques for a 1-D dissipative impact system. The system – a particle, or an ensemble of non-interacting particles, moving in a constant gravitation field and colliding with a varying platform – is described by a nonlinear mapping. The average squared velocity allows to describe the temperature for an ensemble of particles as a function of the parameters using: (i) straightforward numerical simulations; (ii) analytically from the dynamical equations; (iii) using the probability distribution function. Comparing analytical and numerical results for the three techniques, one can check the robustness of the developed formalism, where we are able to estimate numerical values for the statistical variables, without doing extensive numerical simulations. Also, extension to other dynamical systems is immediate, including time dependent billiards.

1. Introduction

In the last decades, modeling of dynamical systems, especially low-dimensional ones, becomes one of the most challenging areas of interest among mathematicians, physicists [1–3] and many other sciences. Depending on both the initial conditions as well as control parameters, such dynamical systems may present a very rich and hence complex dynamics, therefore leading to a variety of nonlinear phenomena. The dynamics can be considered either in the dissipative or non-dissipative regime [4–6] yielding into new approaches, new formalisms therefore moving forward the progress of nonlinear science.

Since the so called Boltzmann ergodic theory [5,6], the assembly between statistical mechanics and thermodynamics has produced remarkable advances in the area leading also to progress in experimental and observational studies [7–11]. Indeed, statistical tools can be used for a complete analysis of the dynamical behavior of such type of systems. Depending on the control parameters, phase transitions and abrupt changes in the phase space can be observed in time as well as in parameter space [6] while many results can be described by using scaling laws approach [12]. In this paper we revisit the 1-D impact system aiming to obtain and describe the behavior of average properties in the chaotic dynamics focusing in the stationary state, *id est*, for very long time, where transient effects are not influencing the dynamics anymore. Analytical expressions will be presented in order to calculate statistical properties for the average velocity, its deviation and the average squared velocity, when these variables reach the stationary state. The developed formalism, allows us to obtain the numerical values for these variables, without doing the numerical simulations. We will show a remarkable agreement between numerical simulations and theoretical analysis considering either statistical and thermal variables, giving so robustness, to the developed theory.

The impact system is described by a free particle, or an ensemble of non-interacting particles, moving under the presence of a constant gravitational field and experiencing collisions with a heavily vibrating platform [13,14]. For elastic collisions, the dynamics leads to a mixed phase space, described in velocity and time, and two main properties are observed according to the control parameter range. If the parameter is smaller than a critical one, invariant spanning curves, also called as invariant tori, are present in the phase space hence limiting the velocity of the particle in a chaotic diffusion for certain portions of the phase space. On the other hand, for a parameter larger than the critical one, invariant spanning curves are not present anymore and unlimited diffusion in velocity, for specific ranges of initial conditions, can be observed. The scenario is totally different when inelastic collisions are considered. In this case, dissipation is in course, hence contracting area in the phase space, therefore leading to the existence

* Corresponding author. Tel.: +55 19 3526 9174.
E-mail addresses: gabriel.diaz.iturry@gmail.com (G. Díaz I.), livorati@rc.unesp.br (A.L.P. Livorati), edleonel@rc.unesp.br (E.D. Leonel).

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of attractors. For strong dissipation and control parameter beyond the critical one, attractors are most periodic. For weak dissipation and large control parameter, chaotic attractors, characterized by a positive Lyapunov exponent [15], dominate over the phase space. Giving the attractors are far away from the infinity (in velocity axis), dissipation has proved to be a powerful way of suppress unlimited diffusion. Because of limited diffusion in phase space, the behavior and properties for both average velocity, average squared velocity or the deviation around the average velocity, known also as roughness, are the following. They grow to start with from a low initial velocity value and, eventually, they bend towards a stationary state [16,17] at very long time. The scenario is scaling invariant with respect to the control parameters and number of collisions with the moving platform. By the use of equipartition theorem, the steady state, obtained in the asymptotic state, can be used to make a connection with the thermal equilibrium of the system [17]. Therefore in the present paper, we evaluate numerically, for long time series, the behavior of: (i) the average velocity; (ii) the averaged squared velocity; and (iii) the deviation around the average velocity, both for the dissipative impact system. We then compare the numerical results with analytical expressions at the equilibrium, obtained via statistical and thermodynamics analysis by using the dynamical equations [17]. A comparison between the results obtained using numerical simulation and theoretical investigation is remarkable, hence giving robustness to the connection between statistical mechanics, thermodynamics and the modeling of dynamical systems. It also improves the theoretical formalism that can be extended to other different types of systems including the time dependent billiards.

The paper is organized as follows: in Sec. 2 we describe the dynamics of the impact system and some of its properties. Section 3 is devoted to the discussion of the numerical investigation. The results using the dynamical equations and connection with the thermodynamics in the stationary state and the discussions of the results are presented in Sec. 4. Finally, Sec. 5 brings some final remarks and conclusions.

2. The model, the mapping and some statistical properties

The model we consider consists of a particle[1] of mass μ moving under the action of a gravitational field and experiences collisions with a heavy periodically moving wall. This model is also referred to as a bouncer or bouncing ball model. It backs to Pustylnikov [18] and has been studied for many years [19–22], with several applications in different areas of research such as vibration waves in a nanometric-sized mechanical contact system [23], granular materials [24–28], dynamic stability in human performance [29], mechanical vibrations [30–32], chaos control [33,34], crises between attractors [35], among many others.

As usual, the dynamics of the system is described by a two-dimensional, non-linear discrete mapping for the variables velocity of the particle v and time t (will be measured latter on as function of the phase of moving wall) immediately after a nth collision of the particle with the moving wall. See Ref. [36] for an analysis as function of the time. The investigations are made based on two main versions of the model: *(i)* complete, which takes into account the whole movement of the vibrating platform; and *(ii)* a static wall approximation. In this version, the nonlinear mapping assumes the wall is static but that, as soon as the particle hits it, there is an exchange of energy as if the wall were moving. This is then a simplified version and shows to be a very convenient way to find out analytical results in the model where transcendental equations do not need to be solved, as they have to be in the complete version. The two versions can be used either to investigate non-dissipative [37] and dissipative dynamics [13,14]. Dissipation here is introduced by using a restitution coefficient $\gamma \in [0,1]$ upon collision. For $\gamma = 1$ the system is non-dissipative albeit area contraction in the phase is observed for $\gamma < 1$.

To construct the mapping, we consider the motion of the platform is described by $y_w(t_n) = \varepsilon \cos wt_n$, where ε and w are, respectively, the amplitude and frequency of oscillation. Moreover, we assume that at the instant t_n, the position of the particle is the same as the position of the moving wall, hence $y_p(t_n) = y_w(t_n)$ and with velocity $V_n > 0$. The mapping then gives the evolution of the states from (V_n, t_n) to (V_{n+1}, t_{n+1}), from (V_{n+1}, t_{n+1}) to (V_{n+2}, t_{n+2}) and so on. To obtain the analytical expressions of the mapping, we have to take into account the time of flight the particle moves without colliding with the wall and, from it, determine the velocity of the moving wall upon collision. From conservation of momentum law we obtain the velocity of the particle after collision. We have indeed four control parameters g, ε, w and γ and not all of them are relevant for the dynamics. Defining dimensionless and hence more convenient variables we have $V_n = v_n w/g$ (dimensionless velocity) and $\epsilon = \varepsilon w^2/g$, which is the ratio between accelerations of the vibrating platform and the gravitational field. We may also measure the time in terms of the number of oscillations of the moving wall $\phi_n = wt_n$. Using this set of new variables, the mapping is written as

$$T_c : \begin{cases} V_{n+1} = -\gamma(V_n^* - \phi_c) - (1+\gamma)\epsilon \sin(\phi_{n+1}) \\ \phi_{n+1} = [\phi_n + \Delta T_n] \mod(2\pi) \end{cases}, \tag{1}$$

where the sub-index c stands for the complete version of the model. The expressions for V_n^* and ΔT_n depend on what kind of collision happens. For the case of multiple collisions, those the particle experiences without leaving the collision zone (a region in space where the moving wall is allowed to move), the corresponding expressions are $V_n^* = V_n$ and $\Delta T_n = \phi_c$ where ϕ_c is obtained from the condition that matches the same position for the particle and the moving wall. It leads to the following transcendental equation that must be solved numerically

$$G(\phi_c) = \epsilon\cos(\phi_n + \phi_c) - \epsilon\cos(\phi_n) - V_n\phi_c + \frac{1}{2}\phi_c^2 . \tag{2}$$

If the particle leaves the collision zone, than indirect collisions are observed. The expressions for the velocity and phase are $V_n^* = -\sqrt{V_n^2 + 2\epsilon(\cos(\phi_n) - 1)}$ and $\Delta T_n = \phi_u + \phi_d + \phi_c$ with $\phi_u = V_n$ denoting the time spent by the particle in the upward direction up to reach the null velocity while the expression $\phi_d = \sqrt{V_n^2 + 2\epsilon(\cos(\phi_n) - 1)}$ corresponds to the time the particle spends from the place where it had zero velocity to the entrance of the collision zone. Finally the term ϕ_c has to be obtained numerically from the equation $F(\phi_c) = 0$ where

$$F(\phi_c) = \epsilon\cos(\phi_n + \phi_u + \phi_d + \phi_c) - \epsilon - V_n^*\phi_c + \frac{1}{2}\phi_c^2 . \tag{3}$$

For the static wall approximation [38], where no transcendental equations must be solved, the mapping has the form

$$T_{swa} : \begin{cases} V_{n+1} = |(\gamma V_n) - (1+\gamma)\epsilon \sin(\phi_{n+1})| \\ \phi_{n+1} = [\phi_n + 2V_n] \mod(2\pi) \end{cases}. \tag{4}$$

The static wall approximation (*swa*), as quoted in the sub-index of mapping (4) is convenient to avoid solving transcendental equations. However, it inherently introduce a new problem that must be taken into account prior evolve the dynamical equations. In the complete version, after a collision with the moving wall, the particle, in specific cases and under certain conditions, can keep moving

[1] Or an ensemble of non-interacting particles.