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# Time-fractional Schamel–KdV equation for dust-ion-acoustic waves in pair-ion plasma with trapped electrons and opposite polarity dust grains ☆

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ABSTRACT

Nonlinear propagation of dust-ion-acoustic (DIA) waves is investigated in a one-dimensional, unmagnetized plasma containing positive ions, negative ions, trapped electrons featuring vortex-like distribution, and immobile dust grains having both positive and negative charges. Via reductive perturbation method, Agrawal's method, and Euler–Lagrange equation, the time-fractional Schamel–KdV equation under the sense of Riesz fractional derivative is derived to describe nonlinear behavior of DIA waves. The approximate solution of the time-fractional Schamel–KdV equation is constructed in terms of Jacobi elliptic functions by variational iteration method. The effect of the plasma parameters on the DIA solitary waves is also discussed in detail.



## 1. Introduction

In recent years, the applications of fractional differential equations have gained increasing attention in plasma physics. Because of the presence of dispersive and/or dissipative forces, the physical processes are non-conservative in the real plasma system [1]. The classical treatments of the forces are the integer-order differential equations, which means these conservative descriptions are not convenient to treat the non-conservative physical processes. It is well-known that the fractional calculus, one of the generalizations of the classical calculus, represents the non-conservative forces and possesses non-local property. Therefore, the fractional differential equations play an important role in describing the non-conservative physical processes in the plasma. For example, El-Wakil and coauthors [2] studied the electron-acoustic solitary waves in a homogeneous system and derived the time-fractional KdV equation. The theoretical results with $\alpha \approx 0.78$ ($\alpha$ is the order of the fractional derivative) are in agreement with the structures of the broadband electrostatic noise observed by Viking satellite in the dayside auroral zone. For more theoretical investigations on the applications of fractional differential equations in plasma physics, see Refs. [3–8].

The pair-ion plasmas, consisting of positive- and negative-charged ions with an equal mass, have become a hot topic because of their wide range of potential applications in astrophysics, space, and laboratory plasmas system [9–12]. Because of the absence of the annihilation, the collective behavior in the pair-ion plasma system can be experimentally studied under controlled conditions. Another feature of pair-ion plasma is that positive and negative ions respond to a potential in the same time scale [9]. In the laboratory, Oohara and Hatakeyama [13] successfully generated the pair-ion plasma consisting of equal mass, positive and negative fullerene ions ($C_{60}^{\pm}$). This work revealed that such plasma system can support three kinds of collective behaviors including low-frequency ion acoustic wave, high-frequency ion plasma wave, and the intermediate frequency wave. After the pioneering experimental investigations on the electrostatic waves in pair-ion plasmas, a number of theoretical works have been presented to study the elementary properties as well as linear and nonlinear collective phenomena in such plasmas [14–17].

It is well-known that the dust grains are ubiquitous in most space and astrophysical plasma system, as well as in the laboratory plasmas. When the dust grains are immersed into a plasma system, they can be charged either negatively or positively depending on the ambient environments [18]. The presence of such extremely massive and highly charged dust grains can modify the characteris-

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tics of the normal waves and introduce new eigenmodes. The dust-ion-acoustic (DIA) wave, as one of the modified dusty modes, was theoretically proposed by Shukla and Silin [19] and experimentally confirmed by Barkan et al. [20]. Since the study of DIA waves can provide new insights into the localized electrostatic perturbations in space and laboratory plasma, considerable research efforts have been devoted to investigating the features of DIA waves in the plasma containing charged dust grains [21–23].

It is indicated that nonlinear propagation of plasma waves is strongly influenced by the velocity distribution functions of electrons. In the past few decades, Maxwellian velocity distribution was the most commonly used distribution. However, in the case of deviation from the isothermality, it is a possibility that groups of electrons have different temperatures where the Maxwellian velocity distribution is not applicable. In such plasma system, the electrons can be trapped and satisfy vortex-like distribution [24, 25]. Several authors [26–30] have already reported that the presence of trapped electrons can introduce a strong nonlinearity into the plasmas and change the structures of nonlinear plasma waves such as solitons, shocks, etc. For example, Alinejad [26] showed that the trapped electrons can support solitary waves with only compressive structures.

The aim of this letter is to investigate the nonlinear propagation of DIA waves by using the time-fractional Schamel–KdV equation in a five-component plasma consisting of positive ions, negative ions, positively charged immobile dust grains, negatively charged immobile dust grains, and trapped electrons featuring vortex-like distribution. In Sec. 2, we present a set of fluid equations for the theoretical model. Using reductive perturbation method, Agrawal's method, and Euler–Lagrange equation, we derive the time-fractional Schamel–KdV equation for DIA waves. In Sec. 3, we solve the time-fractional Schamel–KdV equation by variational iteration method and discuss the effect of the plasma parameters on the DIA solitary waves. Finally, conclusions are given in Sec. 4.

## 2. Theoretical model and time-fractional Schamel–KdV equation

### 2.1. Theoretical model

We consider a one-dimensional, collisionless, and unmagnetized plasma whose constituents are positive ions, negative ions, trapped electrons, and stationary dust grains of opposite polarity (i.e., positively as well as negatively charged dust grains). Because the thermal speed of electrons is much larger than that of ions, we ignore the electron inertia and use the vortex-like distribution function to model the velocity distribution of electrons. In addition, we ignore the dynamics of charged dust grains because they are too heavy to move on the time scale of DIA waves. The dynamics of DIA waves in such plasma system can be described by the following set of normalized continuity, momentum, and Poisson equations.

For positive ions, the normalized fluid equations are

$$\frac{\partial n_p}{\partial t}+\frac{\partial (n_p u_p)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}=-\mu\frac{\partial \Phi}{\partial x}. \tag{2}$$

For negative ions, we have

$$\frac{\partial n_n}{\partial t}+\frac{\partial (n_n u_n)}{\partial x}=0, \tag{3}$$

$$\frac{\partial u_n}{\partial t}+u_n\frac{\partial u_n}{\partial x}=\frac{\partial \Phi}{\partial x}. \tag{4}$$

The above four equations are coupled through the Poisson equation

$$\frac{\partial \Phi^2}{\partial x^2}=\eta n_e+n_n-\beta n_p+\sigma-\delta. \tag{5}$$

The normalized electron number density is

$$n_e=\exp(\Phi)(1-\operatorname{erf}(\sqrt{\Phi}))+\begin{cases}\frac{1}{\sqrt{\gamma}}\exp(\gamma\Phi)\operatorname{erf}(\sqrt{\gamma\Phi}), & \text{for } \gamma\geq 0,\\ \frac{2}{\sqrt{\pi|\gamma|}}\exp(\gamma\Phi\int_0^{\sqrt{-\gamma\Phi}}\exp(X^2)dX), & \text{for } \gamma<0.\end{cases}$$

For $\Phi \ll 1$, the above distribution function can be expanded as

$$n_e=1+\Phi-\frac{4}{3}b\Phi^{\frac{3}{2}}+\frac{1}{2}\Phi^2, \tag{6}$$

where $b=\frac{1}{\sqrt{\pi}}(1-\gamma)$, the parameter $\gamma=T_{ef}/T_{et}$ is the ratio of temperature of free and trapped electrons, $T_{ef}$ and $T_{et}$ are the free and trapped electron temperatures, respectively. Note that $\beta<0$ represents a vortex-like excavated trapped electron distribution. In the above normalized equations, the electric potential $\Phi$, space $x$, and time $t$ are normalized by $\kappa_B T_{ef}/e$, the Debye length $\lambda_D=(\kappa_B T_{ef}/4\pi n_p e^2)^{1/2}$, and the inverse ion plasma frequency $\omega_{pi}^{-1}=(4\pi n_p e^2/m_p)^{-1/2}$, respectively. The number density $n_j$ ($j=p,n$, or $e$ for positive ions, negative ions, or electrons, respectively) is normalized by the equilibrium value $n_{j0}$. And $u_j$ ($j=p$ or $n$) is the fluid velocity of each species normalized by the positive ion sound velocity $C_p=(\kappa_B T_{ef}/m_p)$. Here, $\kappa_B$ is the Boltzmann constant, $e$ the electronic charge, $m_p$ the positive ion mass, $\mu=m_n/m_p$. From the charge neutrality condition at equilibrium, $n_{e0}+n_{n0}+Z_-n_{-0}=n_{p0}+Z_+n_{+0}$, we obtain $1+\eta+\sigma=\beta+\delta$ with $\eta=n_{e0}/n_{n0}$, $\sigma=Z_-n_{-0}/n_{n0}$, and $\delta=Z_+n_{+0}/n_{n0}$. Here, $n_{e0},n_{n0},n_{p0},n_{-0}$, and $n_{+0}$ stand for the equilibrium densities of electrons, negative ions, positive ions, negatively charged dust grains, and positively charged dust grains, respectively.

### 2.2. Time-fractional Schamel–KdV equation

To investigate the small but finite amplitude DIA waves in our plasma system, we employ the reductive perturbation method [31] to Eqs. (1)–(6). We introduce the following stretched coordinates

$$\xi=\epsilon^{\frac{1}{4}}(x-Vt),\quad \tau=\epsilon^{\frac{3}{4}}t, \tag{7}$$

where $0<\epsilon\ll 1$ is a small parameter measuring the weakness of the dispersion, and $V$ is the phase velocity normalized by $C_p$. We expand the dependent variables in the power series of $\epsilon$ as follows

$$\begin{pmatrix}n_p\\n_n\\u_p\\u_n\\\Phi\end{pmatrix}=\begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix}+\epsilon\begin{pmatrix}n_{p1}\\n_{n1}\\u_{p1}\\u_{n1}\\\Phi_1\end{pmatrix}+\epsilon^{\frac{3}{2}}\begin{pmatrix}n_{p2}\\n_{n2}\\u_{p2}\\u_{n2}\\\Phi_2\end{pmatrix}+\cdots \tag{8}$$

Substituting Eqs. (7) and (8) into Eqs. (1)–(6) and collecting the terms in different powers of $\epsilon$, we obtain the following relations for the lowest order in $\epsilon$:

$$n_{p1}=\frac{\mu\Phi_1}{V^2},\quad n_{n1}=-\frac{\Phi_1}{V^2},\quad u_{p1}=\frac{\mu\Phi_1}{V},\quad u_{n1}=-\frac{\Phi_1}{V}. \tag{9}$$

In addition, the phase velocity $V$ can be expressed as

$$V=\pm\sqrt{\frac{\mu+\mu\eta+\mu\sigma-\mu\delta+1}{\eta}}. \tag{10}$$