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Discussion

General band gap condition in one-dimensional resonator-based acoustic metamaterial

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ABSTRACT

A one-dimensional model for resonator-based acoustic metamaterials is introduced. The condition for band gap in such kind of structure is obtained. According to this condition, the dispersion relation is in general a result of the scattering phase and propagating phase. The phenomenon that the band gap is less dependent on lattice structure appears only in the special system in which the coupling between the resonators and the host medium is weak enough. For strong coupled systems, the dispersion of wave can be significantly adjusted by the propagating phase. Based on the understanding, a general guide for band gap optimization is given and the mechanism for structures with the defect states at subwavelength scale is revealed.

1. Introduction

The propagation of acoustic wave in periodic structure has attracted much attention in the past decades. These composite materials are of special interest because they may give rise to acoustic band gaps (BGs). Because the waves inside can be strongly dispersed or completely reflected when their frequencies are close to or inside the BGs, these structures can be used to control the wave-field, and some novel wave phenomena and special utilities, such as the waveguide [1–4], superlens and negative refraction [5, 6], can be realized.

The early studied structure for acoustic BG material, which is named as phononic crystal [7], is constructed by inserting scatterers into a uniform host medium. Because of the structural periodicity, the mechanism of BGs in such kind of structures can be explained naturally as the Bragg scattering of waves. This kind of BGs are usually opened in the frequency region within which the wavelength is comparable to the lattice constant. The followed investigations show that a structure with periodically arranged resonators can also have BGs [8]. Contrasted to BGs in PC structure, the BGs in the resonator-based structure can appear in a low frequency region in which the corresponded wavelength is much greater than the period. Based on this feature, the structures are usually described by the effective medium theory and some abnormal effective parameters, such as negative effective mass density [9] and/or negative effective elastic modulus [10–12], can be obtained. In this sense, the composites are named specially as metamaterials, which means they are not readily observed in natural materials.

To understand the mechanism for such kind of BGs, many investigations have been performed [13–16]. But because most of those researches were concentrated on the systems working at deep subwavelength scale, the retarded phase of the wave in background was usually neglected. As a result, a conclusion that the appearance of BG can be independent on the periodicity of the structure [13–17] was obtained. It can be found however that this conclusion is not valid for all of the structures. For example, it is shown in Ref. [18] that the position of the BGs will be very sensitive to the lattice constant when the resonant frequency of the resonant units goes into a relatively higher frequency region. And in Ref. [3], it is shown that, very similar to the defect mode in phononic crystal, a confined state at subwavelength scale can also be obtained by breaking locally the translational symmetry of the structure. Those phenomena show clearly that the retarded phase of the background wave can change obviously the dispersion of the resonator-based system in some cases.

To give a general understanding of the wave behavior in the resonator-based system, we present a one-dimensional model in which the retarded phase of the wave in background is taken into considered. By this model, we find that the permitted modes in the resonator-based structure can exist only when the summation of the propagating and the scattering phase satisfies a special condition, where the former refers to the retarded phase shift of the

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wave propagating in background and the later is the abrupt phase change when it is scattered by the resonator. Based on these understanding, we not only give a general guide for BG optimization but also reveal the mechanism of the defect modes at subwavelength scale.

2. General condition for one-dimensional resonator-based periodic structure

A typical resonator-based one-dimensional model can be constructed by periodically side-connecting resonators on a one-dimensional uniform host waveguide. It can be a small uniform tube with side-connected Helmholtz cavities for acoustic wave [19], or a metal waveguide with split-ring resonators for electromagnetic wave [20], or a taut string with spring-mass resonators for mechanical wave [21]. By the Bloch theory and transfer matrix method, the dispersion relation for those systems takes the form as

$$\cos qa = \cos ka - \frac{p}{2}\sin ka, \tag{1}$$

where a is the lattice constant, q is the Bloch wave vector, $k = \frac{\omega}{c}$, with ω and c as the angular frequency and the speed of the wave propagating in the uniform host, and p is a parameter related to the impedance ratio between the one-dimensional uniform medium and the resonator. For simplicity, we will take the taut spring with spring-mass resonators as example in the following discussion.

By defining the elastic constant and mass of the resonator as γ and m, and the mass density and the tension of the string as ρ and T, respectively, we can specify the parameter p in Eq. (1) as $p = \frac{\omega a/c}{1-\omega^2/\omega_0^2}\mu$, where $c = \sqrt{T/\rho}$ is the wave speed in uniform background, $\omega_0 = \sqrt{\gamma/m}$ is the resonant frequency of the spring-mass resonator, and $\mu = \frac{m}{\rho a}$ is the mass ratio between the resonator and string, respectively.

One may find that such a dispersion relationship has been solved mathematically in Ref. [21], in which the edge frequencies of the lowest BG are determined by the intersections between the p, $2\cot\frac{\Omega\pi}{2}$ and $-2\tan\frac{\Omega\pi}{2}$ curves, where $\Omega = \frac{\omega a}{c\pi}$ is the non-dimensional frequency. However, it can be found that the physical origin for the BG mechanism is still unclear, even though the BGs can be precisely predicted by their method.

We find that, by the condition $\cos qa \le 1$, which means a propagating wave must has a real Bloch wave vector, a relation between p and ka can be obtained alternatively as

$$\sin(2\alpha + ka)\sin ka \le 0 \tag{2}$$

with

$$\cos\alpha = -\frac{p^2}{\sqrt{p^4+4p^2}},\ \sin\alpha = \frac{2p}{\sqrt{p^4+4p^2}}, \tag{3}$$

where ka is obviously the retarded phase of the wave propagating in the host medium, and α, which will be proved in the follows, is the abrupt phase change when the wave is scattered by the resonator.

To show that α is the scattering phase from the resonator, we need to consider a system with one resonator connected from the left- and right-hand sides by two half-infinite taut strings. For incident waves from left-hand side with harmonic form $y_i = e^{i(kx+\omega t)}$, the scattered wave in the left- and right-hand sides of the resonator can be written as

$$y_{sl} = Se^{i(-kx+\omega t)} \tag{4}$$

and

$$y_{sr} = (1+S)e^{i(kx+\omega t)}, \tag{5}$$

where S is the amplitude of the scattered wave.

By using the continuum condition of the force at the point where the resonator is connected, we have

$$ikT(1-S) = ikT(1+S) + f, \tag{6}$$

where f is the reacting force from the resonator, which satisfies

$$f = m\frac{\partial^2 u}{\partial t^2} = -\gamma[u-(1+S)], \tag{7}$$

where u is the displacement of the mass of the resonator.

By eliminating u in Eq. (7) and then substituting the result into Eq. (6), we can finally get the scattering amplitude

$$S = \frac{ip}{2-ip} = \frac{\sqrt{p^4+4p^2}}{p^2+4}e^{i\alpha}, \tag{8}$$

where α satisfies $\cos\alpha = -\frac{p^2}{\sqrt{p^4+4p^2}}$ and $\sin\alpha = \frac{2p}{\sqrt{p^4+4p^2}}$, which means it is exactly the same one as in Eq. (2).

For system working in the subwavelength scale, we always have $\sin ka > 0$, then Eq. (2) is simplified as

$$\sin(2\alpha + ka) \le 0, \tag{9}$$

which means a harmonic mode is permitted when $(2n+1)\pi \le 2\alpha + ka \le 2(n+1)\pi$ with $n = 0, 1, 2, \cdots$.

From Eq. (9), we can find that the condition for the lowest BG is $2\pi < 2\alpha + ka < 3\pi$, which is the cooperated result of the propagating and the scattering phase. Notice that the conditions $2\alpha + ka = 2\pi$ and $2\alpha + ka = 3\pi$ are mathematically equivalent to $p = 2\cot\frac{\Omega\pi}{2}$ and $p = -2\tan\frac{\Omega\pi}{2}$, respectively, the edges of the lowest BG by our condition will be the same as those by Ref. [21].

To show intuitively the contributions of α and ka to the lowest BG, we give in Fig. 1(a) and (c) the curves of α as functions of the reduced frequency ω/ω_0. For Fig. 1(a), we keep $\omega_0 a/\pi c = 0.5$ to be constant but set $\mu = 0.05$, 0.4 and 1.5, respectively. While for Fig. 1(c), we keep $\mu = 1$ to be constant but set $\omega_0 a/\pi c = 0.05$, 0.3 and 0.5 respectively. To show the BGs, their corresponding curves of $2\alpha + ka$ are also plotted in Fig. 1(b) and (d) respectively, in which BGs are shown by the thin line segments. It can be seen from the figure that generally α increases from $\pi/2$ to π and then to an asymptotic value of $3\pi/2$ when ω increases from 0 to ω_0 and then to infinity respectively. For all of the curves, we have $\alpha = \pi$ at $\omega = \omega_0$. From the curves, it can be easily found that the lower edge of BG will always be reached before ω_0 because of the contribution of ka. As for the upper edge of BG, because $3\pi/2$ is the asymptotic value of α (see the definition of p and α, or the curves in the figure), it can be reached, i.e., the condition $2\alpha + ka = 3\pi$ is satisfied, only when the contribution of ka is taken into considered.

Another interesting phenomenon can be found in the figure is that the shape of α curve is very sensitive to μ and $\omega_0 a/c$. We can see from Fig. 1(a) (Fig. 1(c)) that, by keeping $\omega_0 a/\pi c$ (μ) unchanged but decreasing at the same time μ ($\omega_0 a/\pi c$), the curve becomes sharper and sharper when it crosses through ω_0. To understand this behavior, we need to check the quality factor of a resonator connected to the infinite taut string. By setting $y_i = 0$ and eliminating S in Eqs. (5), (6) and (7), we can get a dispersion relation for the resonator as $\omega^2 - i\frac{\gamma c}{2T}\omega - \omega_0^2 = 0$, which means the behavior of such a resonator can be described by the standard equation $\frac{d^2u}{dt^2} + 2\delta\frac{du}{dt} + \omega_0^2 u = 0$ for the isolated damping resonator. Consequently, the quality factor can be defined as $Q = \frac{\omega_0}{2\delta} = \frac{\omega_0}{\gamma c/2T} = \frac{2}{\mu(\omega_0 a/c)}$. With this analogy and definition, we can understand the phenomenon easily: resonator with a larger quality factor (caused here by smaller $\omega_0 a/\pi c$ or μ) has always a