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Quantum parameter identification for a chaotic atom ensemble system

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ABSTRACT

We propose an indirect method based on an adaptive technique for analyzing the chaos behavior of a general quantum system with complex nonlinear evolution. Using this method, we design an identification function that effectively recognizes the uncertain parameters in a chaotic quantum system only by measuring the system outputs. As an example, we study an atom ensemble in an optical cavity and we obtain a specific parameter identification scheme after analyzing the chaos behaviors. We also verify the accuracy of the identification scheme using numerical simulations and we discuss the influence of different types of errors on the accuracy.

1. Introduction

Chaotic phenomena are ubiquitous in nature and they have extensive applications in classical physics. Since Poincaré first found random-like solutions of deterministic equations [1], chaos theory has developed rapidly in a series of research studies. In particular, the Li–Yorke theorem [2] and Takens' study [3] have described the mathematical characteristics of chaos, as well as providing experimental schemes for applying the chaos effect. In recent years, it has been reported that chaos theory has been applied widely and successfully in the domain of classical physics [4–6].

Consistent with classical physics, in 1982, Fishman et al. indicated that chaos effects may also exist in quantum systems [7]. According to their definition, quantum chaos can be treated as the exponentially rapidly growing distance of quantum phase space trajectories, where the mechanical quantities can exhibit random-like characteristics in the classical limit. Subsequently, chaos behaviors have been found in many quantum systems based on in depth studies, such as cavity quantum electrodynamics (QED) systems [8,9], optomechanical systems [10], gas-phase atoms [11], and spin chain models [12]. In addition to the correspondence with classical dynamical evolution in microcosmic physics, chaos behavior can also have good correlations with some unique properties of quantum mechanics. For instance, Furuya et al. noted that quantum entanglement can reach its maximum in a cavity QED system when chaos effects appear [13]. Later, this conclusion was extended to other entanglement systems, including other quantum correlations [14]. Moreover, it has been proved that quantum chaos will emerge when the quantum systems are in the vicinity of the transition point of a quantum transition, during the quantum squeezing process, or in a quantum decoherence process [15,16]. These phenomena indicate that quantum chaos may be a macroscopic reflection of quantum effects and some quantum effects that are not easy to observe directly can be investigated by analyzing quantum chaos.

In the general case, there are two main explanations for the occurrence of quantum chaos: a significant nonlinear quantum effect exists in the system (e.g., Bose–Einstein condensation [9,17], parametric down-conversion [18], and quantum squeezing [16]), or the parameters of the system become unstable and unpredictable due to the complexity of the system itself and external disturbances. The latter case is usually known as uncertain chaos [19,20]. In contrast to the ideal model, uncertain chaos abounds because complex multi-body effects and coupling with the environment are inevitable in an actual quantum system, which makes it more difficult to analyze uncertain chaos systems than normal systems. In previous studies, quantum chaos has been used to characterize quantum systems in a qualitative manner by establishing a simple homology relationship between chaos and the other properties of the system. However, few studies have investigated chaos quantitatively, and thus systems cannot be described accurately when chaos appears.

In this letter, we propose an indirect method for analyzing quantum chaos. Our main aim is to determine some properties of a system only by considering the chaotic outputs. Therefore, we employ an adaptive technique to design a recognition function, which can effectively avoid the sensitivity and uncertainty of chaotic evolution. After giving the general formalism of the recognition function, we also design a specific scheme as an example

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to demonstrate the capacity for chaos identification in an atom ensemble system. According to subsequent discussion of the implementation conditions, we consider that this scheme is accurate and feasible for use in experiments.

2. Parameter identification

2.1. General formalism

A general dynamic system can always be characterized by some key parameters (p_i), and a correlation can also be established between these parameters and the mechanical quantities (X_i) of the system. For a normal system without chaos, this correlation will be a determined function described by the evolution equations $(\mathcal{D})$, i.e., $\mathcal{D}(p_i) = (X_1, \ldots, X_i)$. In a practical experiment, some unmeasurable parameters can be obtained via the inverse function $p_i = \mathcal{D}^{-1}(X_1, \ldots, X_i)$. However, it is more complicated to realize the idea above when chaos appears, although a similar relationship $\mathcal{C}(p_i) = (X_1, \ldots, X_i)$ can also be obtained by dynamics analysis. Considering that chaotic quantity is a random-like solution that is highly sensitive to the initial conditions and parameters, then $\mathcal{C}^{-1}(X_1, \ldots, X_i)$ does not even exist because of the nonlinear evolution equations.

Fortunately, a chaotic system is not completely uncontrollable. Studies of adaptive techniques have shown that chaotic quantities can achieve synchronization in certain conditions even if each of them is sensitive and irregular. Recently, similar conclusions have also been discussed for quantum systems [21,22], which allows us to propose a new form of parameter identification, i.e., if a defined recognition function where variables are measurable quantities of the system can finally synchronize with the parameter that we want to identify, then we can obtain this parameter instead of analyzing the insoluble $\mathcal{C}^{-1}$.

In this section, we explain how to design a general formula for a recognition function $\tilde{p}(t)$ to satisfy the requirement that $\tilde{p}(t) \to p$ when $t \to \infty$. To achieve this, we define an auxiliary function $V(\tilde{p}, p)$ as follows:

$$V(\tilde{p}, p) = |\tilde{p} - p|^2 \tag{1}$$

We can easily verify that $V \geqslant 0$, and $V = 0$ is tenable only at $\tilde{p} = p$. If we suppose that $\tilde{p}$ and p are both real numbers for simplicity, then Eq. (1) can be further processed as:

$$\dot{V} = 2(\dot{\tilde{p}} - \dot{p})(\tilde{p} - p) = -2\mathcal{P}(t, X_1, \ldots, X_i)[\tilde{p} - p]^2 \tag{2}$$

by setting

$$\dot{\tilde{p}} - \dot{p} = -\mathcal{P}(t, X_1, \ldots, X_i)[\tilde{p} - p]. \tag{3}$$

In this expression, $\mathcal{P}(t, X_1, \ldots, X_i)$ is a designed function that comprises time and the measurable quantities of the system. This is only a mathematical auxiliary function without physical limit, so $\mathcal{P}(t, X_1, \ldots, X_i)$ can be formally taken as a product of the time function $\varphi(t)$ and measurable quantities function $\mathcal{F}(X_1, \ldots, X_i)$ by separating the variables. If $\mathcal{P}(t, X_1, \ldots, X_i) = \varphi(t)\mathcal{F}(X_1, \ldots, X_i)$ is defined as positive definite, then we can ensure that $\dot{V} \leqslant 0$ under the condition of $V \geqslant 0$. In this case, $V(\tilde{p}, p)$ is a so-called Lyapunov function, which can ensure that $\tilde{p}(t) \to p$ with the evolution of time [23]. Thus, Eq. (3) can be treated as the basic configuration of the recognition function. Note that p is a constant that satisfies $\dot{p} = 0$, so we can obtain the final version based on Eq. (3):

$$\dot{\tilde{p}}(t) = -\varphi(t)\mathcal{F}(X_1, \ldots, X_i)[\tilde{p}(t) - p] \tag{4}$$

and Eq. (4) is the basis of the following calculations.

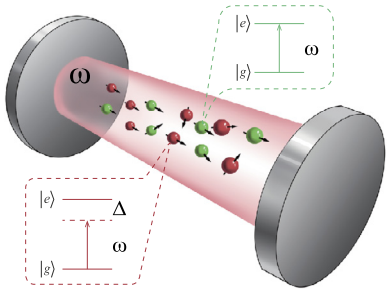

Fig. 1. Two types of atom ensembles coupled with a single-mode radiation field in a resonant cavity. Some atoms (green) interact with the radiation field via a perfect matching frequency ω, whereas others (red) do not interact because of detuning Δ. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2.2. Example based on a cavity QED system

To illustrate our result, we employ a cavity QED system as a straightforward example, which is an interface where light and matter interact. In Fig. 1, a sketch is provided in order to introduce the system considered in a convenient manner. We show that the total number of atoms is N, among which n_1 atoms interact with the radiation field. First, to demonstrate the capacity for parameter identification, we analyze the dynamic evolution of this chaotic system. The total Hamiltonian of our system can be given as follows by neglecting the interactions among atoms:

$$H_{total} = H_F + \sum_{i=1}^{n_1} H_i + \sum_{j=1}^{n_2} H'_j \quad (n_1 + n_2 = N), \tag{5}$$

where $H_F = \omega(a^\dagger a + 1/2)$ is the Hamiltonian of the radiation field (in unit of $\hbar = 1$). Correspondingly,

$$H_i = \frac{1}{2}\omega_i \sigma_{zi} + \lambda\sigma_{xi}(a + a^\dagger) + \beta[a^2 + (a^\dagger)^2 + aa^\dagger + a^\dagger a] \tag{6}$$

and

$$H'_j = \frac{1}{2}\omega_j \sigma'_{zj} \tag{7}$$

are the Hamiltonian of an interacted atom and a free atom. In Eqs. (5)–(7), $a^\dagger$ and a are the creation and annihilation operators of the radiation field, respectively. σ_i refers to the Pauli operator of the ith atom, which satisfies the commutation relation $[\sigma_{xi}, \sigma_{yi'}] = \delta_{ii'} i\sigma_{zi}$. The third term in Eq. (6) describes a feedback effect on the radiation field created by interacting atoms, which will disappear when the atoms do not interact with the field, as shown by Eq. (7) [24].

Using Eq. (5), we can obtain the following Heisenberg equations via $i\dot{c}(t) = [c(t), H_{total}]$, where $c(t)$ refers to the operator in this equation. Then, the operators that describe the radiation field will satisfy:

$$\begin{aligned} &\dot{a} + \dot{a}^\dagger = -i\omega(a - a^\dagger) \\ &\dot{a} - \dot{a}^\dagger = -i(\omega + 4n_1\beta)(a + a^\dagger) - 2i\lambda \sum_{i=1}^{n_1} \sigma_{xi} \end{aligned} \tag{8}$$

and the operator equations of the atoms are:

$$\begin{cases} \dot{\sigma}_{xi} = -\omega_i \sigma_{yi} \\ \dot{\sigma}_{yi} = \omega_i \sigma_{xi} - 2\lambda(a + a^\dagger)\sigma_{zi} \\ \dot{\sigma}_{zi} = 2\lambda(a + a^\dagger)\sigma_{yi} \end{cases}, \quad \begin{cases} \dot{\sigma}'_{xj} = -\omega_j \sigma'_{yj} \\ \dot{\sigma}'_{yj} = \omega_j \sigma'_{xj} \\ \dot{\sigma}'_{zj} = 0 \end{cases} \tag{9}$$

Mean approximation is acceptable in our model because we consider an ensemble that comprises a large number of particles