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Quantum coherence sets the quantum speed limit for mixed states

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ABSTRACT

We cast observable measure of quantum coherence as a resource to control the quantum speed limit (QSL) for unitary evolutions. For non-unitary evolutions, QSL depends on that of the state of the system and environment together. We show that the product of the time bound and the coherence (asymmetry) or the quantum part of the uncertainty behaves in a geometric way under partial elimination and classical mixing of states. These relations give a new insight into the quantum speed limit. We also show that our bound is experimentally measurable and is tighter than various existing bounds in the literature.

1. Introduction

In quantum mechanics, a basic and fundamental goal is to know how to influence a system and control its evolution so as to achieve faster and controlled evolution. Quantum mechanics imposes a fundamental limit to the speed of quantum evolution, conventionally known as quantum speed limit (QSL) [1,2]. With the advent of quantum information and communication theory, it has been established as an important notion for developing the ultra-speed quantum computer and communication channel, identification of precision bounds in quantum metrology [3–5], the formulation of computational limits of physical systems [6–8], the development of quantum optimal control algorithms [9], non-equilibrium thermodynamics [10,11]. The first major result in this direction was put forward by Mandelstam and Tamm [12] in 1945 to give a new perspective to the energy-time uncertainty relation. For pure orthogonal initial and final states evolving under the Hamiltonian H, the bound is given by

$$\tau_{\perp} \geq \frac{\hbar}{\Delta H}. \tag{1}$$

In this paper, we address three basic and fundamental questions. There have been rigorous attempts to achieve more and more tighter bounds and to generalize them for mixed states [13–36]. But we are yet to know (i) what is the ultimate limit of quantum speed? (ii) Can we measure this speed of quantum evolution in the interferometry by measuring a physically realizable quantity? Most of the bounds in the literature are either not measurable in the interference experiments or not tight enough. As a result, they cannot be effectively used in the experiments on quantum metrology, quantum thermodynamics, quantum communication and specially in Unruh effect detection etc., where a small fluctuation in a parameter is needed to detect. Therefore, a search for the tightest yet experimentally realizable bound is a need of the hour [37].

It will be much more interesting, if one can relate various properties of the states or operations, such as coherence, asymmetry, dimension and quantum correlations etc. with QSL. Although these understandings may help us control and manipulate the speed of communication, apart from the particular cases like the Josephson Junction [38] and multipartite scenario [39], there has been little advancement in this direction. Therefore, the third question we ask is: (iii) Can we relate such quantities with QSL? In this paper, we address these fundamental questions and show that quantum coherence or asymmetry plays an important role in setting the QSL.

Quantum coherence on the other hand has taken the central stage in research, especially in quantum biology [40–43] and quantum thermodynamics [44–48] in the last few years. And in quantum information theory, it is a general consensus or expectation that it can be projected as a resource of classically impossible tasks [49–52]. This has been the main motivation to quantify and measure coherence [49,50,53]. Moreover, it is the main resource in the interference phenomenon. Various quantities, such as visibility and various phases in the interferometry are under scanner and the investigation is on to probe various quantum properties or phenomena, such as Unruh effect [54–57], quantum speed limit

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[58], quantum correlation [59] using such quantities in quantum interferometry [60–64]. A proper study of quantum coherence may provide further insight into the development of new techniques to probe such quantum processes in the interferometry.

Here, we consider a new notion of Fubini–Study metric for mixed states introduced in [65]. For unitary evolutions, it is nothing but the Wigner–Yanase skew information [66], which only counts for the quantum part of the uncertainty [52] and a good measure of quantum coherence [50,67] or asymmetry [68–71], which classifies coherence [72] as a resource. Using this metric, we derive a tighter and experimentally measurable Mandelstam and Tamm kind of QSL for unitary evolutions and later generalize for more general evolutions. And thus, we set a new role for quantum coherence or asymmetry as a resource to control and manipulate the evolution speed.

An important question in the study of quantum speed limit may be how it behaves under classical mixing and partial elimination of states. This is due to the fact that this may help us to properly choose a state or evolution operator to control the speed limit. In this paper, we tried to address this question.

In the next section, we introduce the Fubini–Study metric for mixed states along a unitary orbit for our convenience.

2. Metric along unitary orbit

Let $\mathcal{H}_A$ denotes the Hilbert space of the system A. Suppose that the system A with a state $\rho(0)$ evolves to $\rho(t)$ under a unitary operator $U = e^{iHt/\hbar}$. Even if the system is in a mixed state, the purified version of the state must evolve gauge invariantly satisfying the Schrödinger equation of motion. Therefore, the distance between the initial and the final state must be $U(1)$ gauge invariant along the parameter t. To derive such a distance along the unitary orbit, we consider the purification of the state in the extended Hilbert space and define the Fubini–Study (FS) metric for pure states. We know that this is the only gauge invariant metric for pure states. We follow the procedure as in [65] to derive a gauge invariant metric for mixed states from this FS metric for pure states. If we consider a purification of the state $\rho(0)$ in the extended Hilbert space by adding an ancillary system B with Hilbert space $\mathcal{H}_B$ as $|\Psi_{AB}(0)\rangle = (\sqrt{\rho(0)}V_A \otimes V_B)|\alpha\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, the state at time t, must be $|\Psi_{AB}(t)\rangle = (\sqrt{\rho(t)}V_A \otimes V_B)|\alpha\rangle = (U_A\sqrt{\rho}U_A^\dagger V_A \otimes V_B)|\alpha\rangle$, where $|\alpha\rangle = \sum_i |i^A i^B\rangle$ and V_A, V_B are unitary operators on the subsystems A and B respectively. The FS metric for a state $|\psi\rangle$ on the projective Hilbert space can be defined as

$$ds^2_{FS} = \langle d\psi_{projec}|d\psi_{projec}\rangle, \tag{2}$$

where $|d\psi_{projec}\rangle = \frac{|d\psi\rangle}{\sqrt{\langle\psi|\psi\rangle}} - \frac{|\psi\rangle\langle\psi|}{\langle\psi|\psi\rangle^{3/2}}|d\psi\rangle$. This is nothing but the angular variation of the perpendicular component of the differential form $|d\psi\rangle$. The angular variation of the perpendicular component of the differential form for the state $|\Psi_{AB}(t)\rangle$ in this case is given by

$$|d\Psi_{AB_{projec}}(t)\rangle = dt(A_\rho - B_\rho)|\alpha\rangle, \tag{3}$$

where $A_\rho = (\partial_t\sqrt{\rho(t)}V_A \otimes V_B)$, $B_\rho = |\Psi_{AB}(t)\rangle\langle\Psi_{AB}(t)|A_\rho$. Therefore, the FS metric [65] is given by

$$\begin{aligned} ds^2_{FS} &= \langle d\Psi_{AB_{projec}}(t)|d\Psi_{AB_{projec}}(t)\rangle \\ &= dt^2[\langle\alpha|(A_\rho^\dagger A_\rho - A_\rho^\dagger B_\rho - B_\rho^\dagger A_\rho + B_\rho^\dagger B_\rho)|\alpha\rangle] \\ &= \mathrm{Tr}[(\partial_t\sqrt{\rho_t})^\dagger(\partial_t\sqrt{\rho_t})] - |\mathrm{Tr}(\sqrt{\rho_t}\partial_t\sqrt{\rho_t})|^2, \end{aligned} \tag{4}$$

where the second term on the last line becomes zero if monotonicity condition is imposed [65].

Now, suppose that the state of the system is evolving unitarily under $U = e^{iHt/\hbar}$ and at time t, the state $\rho = \rho(t) = U\rho(0)U^\dagger$. We know that square-root of a positive density matrix is unique. If we consider $\rho(0) = \sum_i \lambda_i|i\rangle\langle i|$, then $\rho = \sum_i \lambda_i U|i\rangle\langle i|U^\dagger$ implies $\sqrt{\rho} = \sum_i\sqrt{\lambda_i}U|i\rangle\langle i|U^\dagger = U\sqrt{\rho(0)}U^\dagger$ and uniqueness of the positive square-root implies the uniqueness of the relation. One can show this in an another way by considering arbitrary non-hermitian square-root w of the final state ρ and using the relation $\rho = ww^\dagger = U\rho(0)U^\dagger = U\sqrt{\rho(0)}\sqrt{\rho(0)}U^\dagger = U\sqrt{\rho(0)}V^\dagger V\sqrt{\rho(0)}U^\dagger$, where V is arbitrary unitary operator. Thus, one gets the form of these arbitrary non-hermitian square-roots as $w = U\sqrt{\rho(0)}V^\dagger$. Due to uniqueness of the positive square-root of the positive density matrix, hermiticity condition imposes uniqueness on the arbitrary unitary operators above as $V = U$. Thus, we get $\sqrt{\rho} = U\sqrt{\rho(0)}U^\dagger$, which in turn implies $\frac{\partial\sqrt{\rho}}{\partial t} = \frac{i}{\hbar}[\sqrt{\rho}, H]$. Using this relation and Eq. (4), we get (dropping the subscript FS)

$$ds^2 = -\frac{dt^2}{\hbar^2}[\mathrm{Tr}[\sqrt{\rho}, H]^2] = 2\frac{dt^2}{\hbar^2}Q(\rho, H). \tag{5}$$

The quantity $-[\mathrm{Tr}[\sqrt{\rho}, H]^2] = 2Q(\rho, H)$ in Eq. (5) is nothing but the quantum part of the uncertainty as defined in [52] and comes from the total energy uncertainty $(\Delta H)^2$ on the pure states $|\Psi_{AB}\rangle$ in the extended Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. The quantity is also related to the quantum coherence of the state [50]. By integrating the distance, we get the total distance between the initial state $|\Psi_{AB}(0)\rangle$ and the final state $|\Psi_{AB}(\tau)\rangle$ as

$$s = \int_0^\tau ds = \frac{1}{\hbar}\sqrt{-\mathrm{Tr}[\sqrt{\rho_1}, H]^2}\,\tau, \tag{6}$$

where we have considered the Hamiltonian H to be time independent and $\rho(0) = \rho_1$. Here, we see that the distance between the two pure states on the extended Hilbert space can completely be written in terms of the state ρ_1 and the Hamiltonian $H \in S(\mathcal{H}_A)$, the space of all linear operators belongs to the subsystem A and can also be interpreted as a distance between the initial state ρ_1 and the final state $\rho(\tau) = \rho_2$. Again, we can define the total distance in an another way by considering the Bargmann angle between the initial state and the final state as

$$\begin{aligned} s_0 &= 2\cos^{-1}|\langle\Psi_{AB}(0)|\Psi_{AB}(\tau)\rangle| \\ &= 2\cos^{-1}\mathrm{Tr}(\sqrt{\rho_1}\sqrt{\rho_2}) \\ &= 2\cos^{-1}A(\rho_1, \rho_2), \end{aligned} \tag{7}$$

where the quantity $A(\rho_1, \rho_2) = \mathrm{Tr}(\sqrt{\rho_1}\sqrt{\rho_2})$ is also known as affinity [73] between the states ρ_1 and ρ_2.

3. Quantum speed limits for unitary evolution

Mandelstam and Tamm in [12] showed that the (twice of the) total distance between two pure states measured by integrating the infinitesimal distance from the initial to the final state (6) is greater than the distance defined by the Bargmann angle between the two states as in (7), i.e., $2s \geq s_0$. The inequality, in particular, in this case becomes

$$\tau \geq \frac{\hbar}{\sqrt{2}}\frac{\cos^{-1}A(\rho_1, \rho_2)}{\sqrt{Q(\rho_1, H)}} = \mathcal{T}_l(\rho_1, H, \rho_2). \tag{8}$$

This shows that the quantum speed is fundamentally bounded by the observable measure of quantum coherence or asymmetry of the state detected by the evolution Hamiltonian. If an initial state evolves to the same final state under two different evolution operators, the operator, which detects less coherence or asymmetry in the state slows down the evolution. As a result, it takes more time