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Gate-tunable graphene quantum dot and Dirac oscillator

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ABSTRACT

We obtain the solution of the Dirac equation in $(2+1)$ dimensions in the presence of a constant magnetic field normal to the plane together with a two-dimensional Dirac-oscillator potential coupling. We study the energy spectrum of graphene quantum dot (QD) defined by electrostatic gates. We give discussions of our results based on different physical settings, whether the cyclotron frequency is similar or larger/smaller compared to the oscillator frequency. This defines an effective magnetic field that produces the effective quantized Landau levels. We study analytically such field in gate-tunable graphene QD and show that our structure allows us to control the valley degeneracy. Finally, we compare our results with already published work and also discuss the possible applications of such QD.

1. Introduction

In recent years, several interest have been devoted to the study of two-dimensional (2D) system such as quantum wells, quantum wires, and quantum dots [1–6]. This interest is due to the technological advances in nanofabrication. In addition, one of the most important recent developments in semiconductor has been the achievement of structures in which the electronic behavior is essentially 2D. This means that the charge carriers are confined in a potential such that their motion in one direction is restricted and thus is quantized, leaving only a two-dimensional momentum. In particular, there has been considerable amount of work in recent years on semiconductor confined structures, which finds applications in electronic and optoelectronic devices. The application of a magnetic field perpendicular to the heterostructure plane quantizes the energy levels in the plane, drastically affecting the density of states giving rise to the famous quantum Hall effect [7]. The latter remains as the most interesting phenomenon observed in physics because of its link to different theories and subjects.

Graphene [8,9], two-dimensional crystalline materials, has become one of the most important subjects in condensed matter research in the last few years. This new material has a number of unique properties, which makes it one of the most promising materials for future nanoelectronics [10]. One of them is the band structure, which is gapless and exhibits a linear dispersion relation at two inequivalent points K and K' in the vicinity of the Fermi energy. Moreover, its low energy of electrons is governed by a $(2+1)$-dimensional Dirac equation. Those electrons behave as massless chiral fermions, i.e. relativistic electrons. Consequently, the electrons cannot be localized by any confinement potential, which is related to the fact that electrons in graphene can have both positive and negative energies, i.e. Klein tunneling effect [11]. Unfortunately, the Dirac fermions cannot be confined by electrostatic potentials. This is due the so-called Klein tunneling effect [11] and the absence of the gap in the spectrum. Thus the realization of the quantum dots (QD) is needed to overcome such situation. Recently, alternative strategies have been proposed to confine charged particles by using thin single-layer graphene strips [12,13] or nonuniform magnetic fields [14].

On the other hand, the Dirac oscillator was proposed by Moshinsky and Szczepaniak [15] in 1989 and is considered as the relativistic version of the harmonic oscillator. The Dirac oscillator has been studied extensively [16–21] because of their probable applications in many branches of physics. Additionally, the Dirac oscillator has been used in optics [22] and Jaynes Cummings model [23]. It is only recently that the first experimental microwave realization of the one-dimensional Dirac oscillator was developed [24]. The experiment relies on a relation of the Dirac oscillator to a corresponding tight-binding system. Later on, it is shown that the two-dimensional Dirac oscillator model can describe some properties of electrons in graphene [25]. This model was used to explain the origin of the left-handed chirality observed for charge carriers in monolayer and bilayer graphene.

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We combine different approaches to achieve our goal. Indeed, based on [26,27] we set the Hamiltonian system of our problem where a similarity transformation is used to simplify the process for obtaining the solutions. Later on, we define the QD by gates introducing an electrostatic confining potential. We find the bound state solution of gate-tunable graphene QD in the presence of a constant magnetic field B and Dirac oscillator of frequency ω as well as a mass term that might be introduced by the underlying substrate [28,29]. The eigenspinors are obtained in terms of the confluent hypergeometric functions showing dependence of B and the oscillator coupling ω.

Subsequently, we analyze the impact of the external field B on the solutions of the energy spectrum of the QD by extracting interesting properties. More precisely, we consider three different cases corresponding to the relative strength of B with respect to ω. In doing so, we start by defining an effective magnetic field, that produces the effective quantized Landau levels, and focus on its dependence of the bound states in circularly symmetric QD. We show how to control the valley degeneracy by manipulating the effective magnetic field. This can help to form the valley filters, valves [30], or qubits [31], and spin qubits [32] in graphene.

The paper is organized as follows. In section 2, we set our problem by reviewing some mathematical tools needed to deal with our issues. To investigate the basic feature of the gate-tunable graphene QD, we set the appropriate confining potential and give the corresponding solutions of the energy spectrum in section 3. Using the matching condition at the boundary, we obtain the condition that governs the existence of the bound state. This will serve to study different liming cases related to the strength of the magnetic field. We conclude our results in the final section.

2. Theoretical model

In order to deal with our task we establish an appropriate Dirac equation describing our system. To go deeply in our study for the graphene QD, we introduce a mass term to open a gap.

2.1. Dirac equation

To start let us set some mathematical background related to Dirac formalism needed to deal with our task. Indeed, a particle of mass m in the presence of a constant perpendicular magnetic field can be described by considering the Dirac equation in $(2+1)$ dimensions

$$\left[i\gamma^{\mu}(\partial_{\mu}+iA_{\mu})-m\right]\psi=0, \qquad \mu=0,1,2 \tag{1}$$

where the electromagnetic potential $A_{\mu}=(A_0,\vec{A})$ and the space–time gradient $\partial_{\mu}=\left(\frac{\partial}{\partial t},\vec{\nabla}\right)$. Here we have the representation $\gamma^0=\sigma_3$ and $\vec{\gamma}=i\vec{\sigma}$ with the 2×2 hermitian Pauli spin matrices $\{\sigma_i\}_{i=1}^{3}$. The Dirac matrices γ^{μ} satisfy the algebra

$$\left\{\gamma^{\mu},\gamma^{\nu}\right\}=\gamma^{\mu}\gamma^{\nu}+\gamma^{\nu}\gamma^{\mu}=2\mathcal{G}^{\mu\nu}, \qquad \mu,\nu=0,1,2 \tag{2}$$

with the metric $\mathcal{G}=\mathrm{diag}(+\ -\ -)$.

In our study of system made of graphene, we need to consider massless Dirac fermions. To this end, we multiply (1) by σ_3 to open a gap, such as

$$i\frac{\partial}{\partial t}\psi=\left(-i\vec{\alpha}\cdot\vec{\nabla}+\vec{\alpha}\cdot\vec{A}+A_0+m\beta\right)\psi \tag{3}$$

where we have set $\vec{\alpha}=i\sigma_3\vec{\sigma}$ and $\beta=\sigma_3$. For time independent potentials, the two-component spinor wavefunction is separable $\psi(t,r,\theta)=e^{-iEt}\psi(r,\theta)$. For regular solutions of (3), square integrability and the boundary conditions require that $\psi(r,\theta)$ satisfies

$$\sqrt{r}\,\psi(r,\theta)\big|_{\substack{r=0\\ r\to\infty}}=0, \qquad \psi(\theta+2\pi)=\psi(\theta). \tag{4}$$

To simplify the construction of the solution, we look for a local 2×2 similarity transformation $\Lambda(r,\theta)$ that maps the cylindrical projection of the Pauli matrices $(\vec{\sigma}\cdot\hat{r},\ \vec{\sigma}\cdot\hat{\theta})$ into their canonical Cartesian representation (σ_1,σ_2), respectively. That is

$$\Lambda\,\vec{\sigma}\cdot\hat{r}\,\Lambda^{-1}=\sigma_1, \qquad \Lambda\,\vec{\sigma}\cdot\hat{\theta}\,\Lambda^{-1}=\sigma_2. \tag{5}$$

We note that any other choice for the pair of Pauli matrices can be obtained from the present one through a unitary transformation, hence leaving the physics of the problem unaltered. A 2×2 matrix that is defined by [26,27]

$$\Lambda(r,\theta)=\frac{1}{\sqrt{r}}\,e^{\frac{i}{2}\sigma_3\theta}. \tag{6}$$

We are interested in the Dirac oscillator for its probable application in many branches of physics as we noticed before [22,24,25]. Motivated by these investigations, we consider such oscillator in another context and emphasize its influence on a system based on the QD. To achieve this goal, we introduce an additional coupling as the 2D Dirac-oscillator potential [15,33], that keeps symmetry of the system. This coupling is introduced by the substitution $\vec{\nabla}\to\vec{\nabla}+\lambda\omega\vec{r}\beta$ where ω is the oscillator frequency and λ is a constant parameter. To simplify the forthcoming analysis, we require that the condition $\lambda=m$ should be fulfilled. Now from the above consideration, we obtain the $(2+1)$-dimensional Dirac equation for a charged spinor in static electromagnetic potential

$$(H-E)\,\chi_{\pm}=0 \tag{7}$$

where the Hamiltonian is given by

$$H=\begin{pmatrix}0 & \partial_r+iA_r-\lambda\omega r-\frac{i}{r}\partial_\theta+A_\theta\\ -\partial_r-iA_r-\lambda\omega r-\frac{i}{r}\partial_\theta+A_\theta & 0\end{pmatrix}+\lambda\sigma_3+A_0\mathbb{I} \tag{8}$$

and $\chi_{\pm}$ are the components of the transformed wavefunction $|\chi\rangle=\Lambda\,|\psi\rangle$, with Λ given in (6). It is clearly seen that the second term is gap and third one can be regarded as an external potential. In the forthcoming analysis, we will fix different potential in order to deal with some basic features some properties of the gate-tunable graphene QD.

2.2. Mass term

Usually, the charge carriers in graphene have no rest mass. There are different ways how a mass term can be introduced and realized in graphene, one may see [34,35]. Motivated by these works, we want to show that our model is sharing some common features with graphene systems. This can be done by treating the mass term appearing in our model as an opining gap. Our statement will be the subject of the forthcoming analysis.

To study the impact of the external field B and oscillating frequency ω we consider a system made of graphene described by the Hamiltonian (8) except we replace $\lambda\sigma_3$ by $\tau\lambda\sigma_3$ with $\tau=\pm 1$ differentiates the two valleys K and K'. Thus, we call the wave function $\chi_{\pm}^{\tau}$ spinor. To go further, let us set some quantities such a constant magnetic field of strength B applied perpendicular to the (r,θ)-plane, which is $\vec{B}=B\,\hat{z}$. We choose the gauge $\vec{A}(r,\theta)=\frac{B}{2}(0,r)$ and assume a circular symmetry in the confinement potential $A_0=U(r)$. Consequently, (7) becomes completely separable in radial and angular parts. Then, we can write the spinor wavefunction as

$$\chi_{\pm}^{\tau}(r,\theta)=\phi_{\pm}^{\tau}(r)\,\varphi(\theta) \tag{9}$$

such that the angular component satisfies the eigenvalue equation $-i\frac{d\varphi}{d\theta}=\xi\,\varphi$ where ξ is a real separation constant giving the function

$$\varphi(\theta)=\frac{1}{\sqrt{2\pi}}e^{i\xi\theta}. \tag{10}$$