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Dynamics of neutral atoms in artificial magnetic field

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ABSTRACT

Cyclotron dynamics of neutral atoms in a harmonic trap potential with artificial magnetic field is studied theoretically. The cyclotron orbit is obtained analytically and confirmed numerically. When the external harmonic potential is absent, artificial magnetic field can result in the singly periodic circular motion of Bose gas with the emergence of a Lorentz-like force, which is similar to particles with electric charge moving in a magnetic field. However, the coupling between artificial magnetic field and harmonic trap potential leads to rich and complex cyclotron trajectory, which depends on $\sqrt{B^2+1}$, where B is the rescaled artificial magnetic field. When $\sqrt{B^2+1}$ is a rational number, the cyclotron orbit is multiply periodic and closed. However, when $\sqrt{B^2+1}$ is an irrational number, the cyclotron orbit is quasiperiodic, i.e., the cyclotron motion of Bose gas is limited in a annular region, and eventually, the motion is ergodic in this region. Furthermore, the cyclotron orbits also depend on the initial conditions of Bose gas. Thus, the cyclotron dynamics of Bose gas can be manipulated in a controllable way by changing the artificial magnetic field, harmonic trap potential and initial conditions. Our results provide a direct theoretical evidence for the cyclotron dynamics of neutral atoms in the artificial gauge field.

1. Introduction

Within less than a decade, ultracold quantum gases have successfully pervaded many fields of physics. This is particularly true in the condensed matter realm where ultracold atoms become a key player in many-body physics [1]. Recently, the realization of artificial gauge fields in neutral bosonic atoms based on the Berry phase effect [2] and its non-Abelian generalization has attracted great interests [3–6]. When a neutral atom moves in a properly designed laser field which acts on an artificial gauge magnetic field, its center-of-mass motion may mimic the dynamics of a charged particle in a magnetic field, with the emergence of a Lorentz-like force [4,7]. These fictitious gauge fields are produced by an all optical Raman process, and can generate some interesting phenomena, such as the creation of vortices in the superfluid state of bosons [4], the oscillations of atomic clouds driven by artificial electric field [8], the transport properties [9,10], the phase transitions [11–14], and the nonlinear waves [15–17]. The creation of vortices without rotating the atomic cloud demonstrates the presence of the artificial magnetic field. In particular, the quantum cyclotron dynamics of a single neutral atom restricted to a four-site plaquette is observed in recent experiments [18,19], which is also a direct evidence for the existence of artificial gauge magnetic field. However, the researches of this cyclotron dynamics of neutral atoms are only limited to discrete system [20–23], thus it would be an important and interesting issue to theoretically understand cyclotron dynamics of neutral atoms in continuous system. Here, we propose a theoretical scheme for observation of the cyclotron dynamics in a harmonic trap potential and a free space, which can be realized more easily in the experiment.

In this paper, we concentrate on this topic and provide a deep insight into the cyclotron dynamics of neutral atoms in a harmonic trap potential with the artificial magnetic field. The cyclotron orbit is obtained analytically and confirmed numerically. Without the harmonic trap potential, artificial magnetic field can result in the singly periodic circular motion of the neutral atoms with the emergence of a Lorentz-like force, which is similar to particles with electric charge moving in a magnetic field. However, the coupling between artificial magnetic field and harmonic trap potential leads to the cyclotron trajectory of Bose gas presenting some interesting and complex structures, which depends on $\sqrt{B^2+1}$. $B = B_e/(m\omega)$ is the dimensionless artificial gauge magnetic field, where B_e is the effective artificial gauge magnetic field, ω is the frequency of external harmonic trap potential, and m is atomic mass. The dimensionless artificial gauge magnetic field can be manipulated by appropriately adjusting the ratio between effective artificial magnetic field and the frequency of external harmonic trap potential. When $\sqrt{B^2+1}$ is a rational number, the cyclotron orbit is multi-

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ply periodic and closed. However, when $\sqrt{B^2+1}$ is an irrational number, the cyclotron motion is quasiperiodic, i.e., the cyclotron motion of Bose gas is limited in an annular region, and eventually, the motion is ergodic in this region. These cyclotron orbits also depend on the initial position and momenta of the wave packet. Thus, the cyclotron dynamics of Bose gas can be manipulated in a controllable way by changing the artificial magnetic field, harmonic trap potential and initial conditions.

2. Model and variational approach

We focus on the effect of uniform artificial magnetic field on ultracold atomic Bose gas trapped in a two-dimensional harmonic trap potential. Then, the system can be well described by a dimensionless 2D Gross–Pitaevskii equation [4,7],

$$\mathrm{i}\frac{\partial\psi}{\partial t}=\left[\frac{1}{2}(\mathbf{P}-\mathbf{A})^2+V+g|\psi|^2\right]\psi, \tag{1}$$

where $\mathbf{P}=-\mathrm{i}\nabla$ is the momentum operator. ψ is the macroscopic wave function of the condensate normalized so that $\int|\psi|^2dxdy=1$. The physical variables are rescaled as $\psi\sim(l_xl_y)^{-1/2}\psi$, $t\sim\omega^{-1}t$, $x\sim l_xx$, $y\sim l_yy$ with $l_x=\sqrt{\hbar/m\omega_x}$ and $l_y=\sqrt{\hbar/m\omega_y}$ being the characteristic length of the harmonic oscillator in the x and y direction, respectively, in which m is atomic mass, ω_x and ω_y is the trapping frequency of the harmonic oscillator in the x and y direction, respectively, here, we set $\omega_x=\omega_y=\omega$. $g=N\hbar a_s/(\sqrt{\pi}\omega l_xl_yl_z)$ is the effective interatomic interaction with l_z being the characteristic length of the harmonic oscillator in the z direction, N being the total particle number trapped in the harmonic trap potential and a_s being the interatomic s-wave scattering length. The harmonic trap potential $V=\frac{1}{2}\lambda(x^2+y^2)$, here, $\lambda=0,1$, which denotes without or with harmonic trap potential, respectively. Here, we choose the symmetrical gauge for the vector potential $\mathbf{A}(\mathbf{r})=(By,-Bx,0)$, and the artificial gauge magnetic field associated with $\mathbf{A}(\mathbf{r})$ is a uniform magnetic field $\mathbf{B}=2B\mathbf{e_z}$ in the z direction. $B=B_e/(m\omega)$ is the dimensionless artificial gauge field, where B_e is the effective artificial gauge magnetic field. This artificial magnetic field results in the cyclotron motion of neutral atom with the emergence of a Lorentz-like force, which is similar to a charged particle moving in a magnetic field [4,7]. Generally, the effective magnetic field B_e depends only on the geometry of the coupling between internal states of particles, and can be controlled by adjusting the noncollinear gradients of the spatial variation of mixing angle and laser phase angle [7].

To obtain the artificial gauge field induced cyclotron orbit of Bose gas, we use a variational method corresponding to the minimization of the action related to the Lagrangian

$$L=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\frac{\mathrm{i}}{2}(\psi\frac{\psi^*}{\partial t}-\psi^*\frac{\psi}{\partial t})+V|\psi|^2+\frac{g}{2}|\psi|^4+\frac{1}{2}\psi^*(\mathbf{P}-\mathbf{A})^2\psi\right\}dxdy, \tag{2}$$

where the asterisk denotes a complex conjugate. In order to obtain the dynamics of the condensate in the trapping potential we will find the extremum of Eq. (2) with a set of trial functions. In our case, a natural choice of the trial function is a Gaussian, since in the absence of interactions it is precisely the ground state of the linear Schödinger equation and interatomic interactions only change the dimensions of atomic cloud [24]. Although a Gaussian trial wave function is not unique, but it is the simplest and most convenient for this variational analysis, and it can guarantee the trial function corresponding to the ground state wave function, so we take

$$\psi=\frac{\mathrm{Exp}\{\sum_{j=x,y}[-\frac{(j-\xi_j)^2}{2w_j^2}+\mathrm{i}p_j(j-\xi_j)+\mathrm{i}\frac{\delta_j}{2}(j-\xi_j)^2]\}}{\sqrt{\pi w_xw_y}}. \tag{3}$$

At a given time t, this function defines a Gaussian distribution centered at the position $(\xi_x(t),\xi_y(t))$ with width $(w_x(t),w_y(t))$, momenta $(p_x(t),p_y(t))$, and the variational rate of width $(\delta_x(t),\delta_y(t))$. Note that, when the interatomic interaction and the total particle number trapped in the harmonic trap potential are large enough to make the kinetic energy negligible, the wave function of condensate is closer to the Thomas–Fermi approximation solution with parabolic dependence than to a Gaussian [24,25], thus, this variational approach is not valid under the condition of large particle number and strong interatomic interaction. Inserting Eq. (3) into Eq. (2) and applying the Euler–Lagrangian equations $\frac{\partial L}{\partial q_i}-\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}=0$, where $q_i=\{\xi_x,\xi_y,w_x,w_y,p_x,p_y,\delta_x,\delta_y\}$, we obtain

$$\ddot{\xi}_x+\lambda\xi_x+2B\dot{\xi}_y=0, \tag{4}$$

$$\ddot{\xi}_y+\lambda\xi_y-2B\dot{\xi}_x=0, \tag{5}$$

$$p_x=\dot{\xi}_x+B\xi_y, \tag{6}$$

$$p_y=\dot{\xi}_y-B\xi_x, \tag{7}$$

$$\ddot{w}_x+(\lambda+B^2)w_x=\frac{1}{w_x^3}+\frac{g}{2\pi w_x^2w_y}, \tag{8}$$

$$\ddot{w}_y+(\lambda+B^2)w_y=\frac{1}{w_y^3}+\frac{g}{2\pi w_y^2w_x}, \tag{9}$$

$$\delta_x=\frac{\dot{w}_x}{w_x}, \tag{10}$$

$$\delta_y=\frac{\dot{w}_y}{w_y}. \tag{11}$$

Therefore, once we know the behavior of center and width of the condensate, we can calculate the evolution of the rest of the parameters, and then completely characterize the dynamics of the condensate. Equations (4)–(11) indicate that, the dynamics of the wave packet center in the x and y directions is coupled by artificial magnetic field B. The coupling induced by B suppresses the diffusion of the wave packet and therefore coherently traps the Bose gas even if without external harmonic potential (i.e., $\lambda=0$). Interestingly, the dynamics of the wave packet center has nothing to do with the interatomic interaction, and the interatomic interaction only influences the breathing dynamics of the wave packet width. Thus, in order to clearly demonstrate the cyclotron dynamics, we only consider the case without interatomic interaction.

Linearizing Eqs. (8) and (9) around the equilibrium points (w_{x0},w_{y0}), we obtain the breathing mode of the condensate width along x and y directions

$$\Omega_{x,y}^2=\lambda+B^2+\frac{3}{w_{x0,y0}^4}+\frac{g}{\pi w_{x0}w_{y0}w_{x0,y0}^2}. \tag{12}$$

Obviously, the breathing mode of the wave packet width has been influenced by harmonic trap potential, artificial magnetic field and interatomic interaction. This is illustrated in Fig. 1. The oscillating frequency of the wave packet width is blue shifted by the artificial magnetic field and interatomic interaction. The rate of the frequency shift increases with magnetic field (see Fig. 1(a)), but decreases with interatomic interaction (see Fig. 1(b)). For fixed artificial magnetic field and interatomic interaction, the harmonic trap potential also results in the blue frequency shift. The frequency shift induced by the harmonic trap potential is more significant when the artificial magnetic field is weak. In a word,