Physics Letters A 380 (2016) 525–529

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Multi-flexural band gaps in an Euler–Bernoulli beam with lateral local resonators

Ting Wang [a,b], Mei-Ping Sheng [a], Qing-Hua Qin [b]

[a] School of Marine Science and Technology, Northwestern Polytechnical University, Xi'an, Shaanxi, 710072, PR China
[b] College of Engineering and Computer Science, The Australian National University, ACT, 2600, Australia

ARTICLE INFO

Article history:
Received 30 September 2015
Received in revised form 1 December 2015
Accepted 2 December 2015
Available online 10 December 2015
Communicated by R. Wu

Keywords:
Flexural vibration suppression
Lateral local resonator
Band gaps
Wave transformation
Frequency response function

ABSTRACT

Flexural vibration suppression in an Euler–Bernoulli beam with attached lateral local resonators (LLR) is studied theoretically and numerically. Hamilton's principle and Bloch's theorem are employed to derive the dispersion relation which reveals that two band gaps are generated. Within both band gaps, the flexural waves are partially transformed into longitudinal waves through a four-link-mechanism and totally blocked. The band gaps can be flexibly tuned by changing the geometry parameter of the four-link-mechanism and the spring constants of the resonators. Frequency response function (FRF) from finite element analysis via commercial software of ANSYS shows large flexural wave attenuation within the band gaps and the effect of damping from the LLR substructures which helps smooth and lower the response peaks at the sacrifice of the band gap effect. The existence of the multi-flexural band gaps can be exploited for the design of flexural vibration control of beams.

1. Introduction

Investigations on wave propagation in periodic structures have received much attention in recent years [1–4]. The studies are focused on the generating of unique band gaps within which acoustic waves are totally attenuated. Their related applications are promising in vibration isolators, frequency filters and waveguides. Previous configurations proposed in Refs. [5–7] are mainly on one-dimensional lattice for controlling the longitudinal wave behaviour, which is far from practical application.

Beams are widely used in engineering constructions. Waves propagating through beams may cause damages for the structures and inaccuracy for some experimental measurements. Several structural configurations using the band gap concept have been designed for the control of the behaviour of waves in beams. In the configurations, the local resonators are attached to continuum beams to generate band gaps for stopping the propagation of waves, including longitudinal wave [8], flexural wave [9–11] and torsional wave [12]. As for practical engineering applications, the control of flexural wave is of great importance for structures working under water regarding their radiation safety. With this awareness, Yu et al. [13] and Liu et al. [14] investigated flexural wave in different types of beams to prevent its propagation, which provides guidance in vibration suppression design. Sun et al. [15] attached small spring–mass–damper subsystems to a uniform isotropic beam to form a metamaterial, aiming at band gap generation for flexural vibration absorption.

Beams mentioned above yield only single band gap, which is inapplicable to devices or cases requiring multi-flexural wave suppression. Besides, design and modelling of beams having multi-flexural band gaps is more difficult due to their higher DOFs. Few researches have been carried out on this. So far, Wen et al. [16] and Wang et al. [17] designed a multi-band gap beam by attaching multi-local resonators periodically to a beam based on previous single band gap concept. Their work paved a way for multi-wave suppression. Pai later extended their previous work [18], developing modelling and analysis methods to reveal the actual working mechanism of the multi-band gaps metamaterial beam for absorption of low frequency waves. All the work they've done focuses on the attachment of multi-resonators for the flexural wave control. No one has transformed the flexural waves to longitudinal waves and attenuated the flexural vibration in another direction in a beam.

Inspired by the LLR configuration proposed by Huang and Sun [19], this paper proposes a new metamaterial beam to generated multi-flexural band gaps with LLR substructures attached. The LLR structures can partially transform the flexural waves into longitudinal waves, and block the wave propagation in another direction. The rest of this letter is organised as follows. A concise derivation of the Hamilton's principles of the LLR beam is provided in Section 2 and validated using the finite element method in Section 3

E-mail address: WT323@mail.nwpu.edu.cn (T. Wang).

http://dx.doi.org/10.1016/j.physleta.2015.12.010
0375-9601/

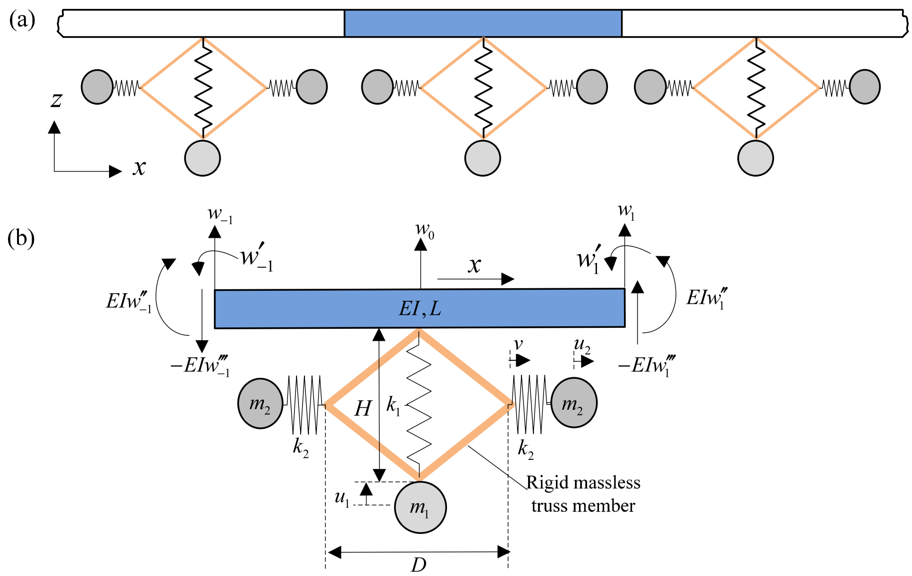

Fig. 1. Construction of metamaterial beam: (a) an infinite beam, (b) a typical unit cell.

[20], with the analysis of the effect of the geometry parameters and damping on the band gaps. Design of the multi-band gap beams in applications is presented in the section of conclusion.

2. Theory and modelling

Fig. 1 shows a simple model of an Euler–Bernoulli beam with periodical LLR substructures in x direction. One LLR consists of two lateral resonators with spring and mass constant of k_2 and m_2, a vertical resonator with spring and mass constant of k_1 and m_1, and a four-link-mechanism with rigid and massless trusses. The beam and the vertical resonator vibrate in z direction and the lateral resonators vibrate in x directions, with displacements of w, u_1 and u_2, respectively. The vertical distance and the horizontal distance of the four-link-mechanism are H and D. The length of the unit cell is L, and A, I, E and ρ denote the beam's cross-section area, area moment, Young's modulus and mass density, respectively. The dispersion relation is derived below.

The governing equation for a unit cell of an infinite periodic metamaterial beam can be obtained by using the extended Hamilton's principle

$$\int_0^L (\delta T - \delta U + \delta W_{nc})dt = 0 \tag{1}$$

where K is the kinetic energy, U is the elastic energy, and W_{nc} is the non-conservative work from the external loads. From the typical unit cell, every item in Eq. (1) follows

$$\delta T = -\int_{-L/2}^{L/2} \rho A\ddot{w}\delta w dx - m_1\ddot{u}_1\delta u_1 - 2m_2\ddot{u}_2\delta u_2 \tag{2}$$

$$\begin{aligned}\delta U = &\int_{-L/2}^{L/2} EIw''\delta w'' dx + k_1(u_1 - w_0)\delta(u_1 - w_0)\\ &+ 2k_2(u_2 - v)\delta(u_2 - v)\\ = &\int_{-L/2}^{L/2} EIw^{(4)}\delta w dx + k_1(u_1 - w_0)\delta(u_1 - w_0)\\ &+ 2k_2(u_2 - v)\delta(u_2 - v)\\ &+ EI\big(w_1''\delta w_1' - w_{-1}''\delta w_{-1}' - w_1'''\delta w_1\\ &+ w_{0+}'''\delta w_0 - w_{0-}'''\delta w_0 + w_{-1}'''\delta w_{-1}\big)\end{aligned} \tag{3}$$

$$\delta W_{nc} = EIw_1''\delta w_1' - EIw_{-1}''\delta w_{-1}' + EIw_{-1}'''\delta w_{-1} - EIw_1'''\delta w_1 \tag{4}$$

Based on the assumption of small displacements, we have

$$v = -\frac{H}{2D}(w_0 - u_1) \tag{5}$$

where w_0 represents the flexural displacement of the centre of the beam, and v is the displacement of the truss end connected the lateral resonators. Substitution Eqs. (2)–(5) into Eq. (1) yields

$$\begin{aligned}0 = \int_0^t \Bigg\{ &\int_{-L/2}^{L/2} \Bigg\{ -\rho A\ddot{w} - EIw^{(4)}\\ &+ \Bigg[k_1(u_1 - w_0) - \frac{H}{D}k_2\left(u_2 + \frac{H}{2D}(w_0 - u_1)\right)\\ &+ EI\big(w_{0+}'''\delta w_0 - w_{0-}'''\delta w_0\big)\Bigg]\delta(x)\Bigg\}\delta w dx\\ &+ \Bigg[-m_1\ddot{u}_1 - k_1(u_1 - w_0)\\ &+ \frac{H}{D}k_2\left(u_2 + \frac{H}{2D}(w_0 - u_1)\right)\Bigg]\delta u_1\\ &+ \Bigg[-2m_2\ddot{u}_2 - 2k_2\left(u_2 + \frac{H}{2D}(w_0 - u_1)\right)\Bigg]\delta u_2\Bigg\}dt\end{aligned} \tag{6}$$

where $\delta(x)$ is the Dirac function, $w' = \partial w/\partial x$ and $\dot{w} = \partial w/\partial t$. Moreover, $EIw_{0-}''' \neq EIw_{0+}'''$ because of a concentrated shear force created by the LLR substructure at $x = 0$. By setting the coefficients of δw, δu_1 and δu_2 in Eq. (6) to zero, the governing equations can be obtained.

$$-\rho A\ddot{w} - EIw^{(4)} + \left[k_1(u_1 - w_0) - \frac{H}{D}k_2\left(u_2 + \frac{H}{2D}(w_0 - u_1)\right)\right.$$