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Characteristic entanglement timescales of a qubit coupled to a quartic oscillator

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ABSTRACT

The structure of the entanglement dynamics of a qubit coupled to a quartic oscillator is investigated through the calculation of timescales of visibility and predictability, and their relation with the concurrence dynamics. This model can describe a Rydberg atom in a Kerr medium. A method based on the analysis of the different interference processes of the terms that compose the physical quantities studied is proposed, and timescales related to decay, revivals and fast oscillations under the decay envelope are computed. The method showed to be effective for the vast majority of cases studied, even when the timescales vary several orders of magnitude. The conditions for expansions in power series to give correct decay timescales are analyzed.

1. Introduction

The study of dynamical systems is extremely important in many fields, such as mathematics, physics, biology, economics, engineering, geology and medicine. Characteristic timescales are often employed for the characterization of the dynamics, frequently aiming at applications. For a periodic system, the period is the main timescale, whereas for a chaotic system, such role is played by the inverse of the Lyapunov exponent. An ensemble of classical trajectories, on the other hand, has a characteristic timescale associated to the collapse of the dynamics. An important discussion in statistical physics is related to the time for the system to converge to the Boltzmann–Gibbs distribution [1–6]. In a recent contribution, Parolo and co-workers [7] investigated the characteristic time in which scientific publications have relevant impact: they observed that it scales inversely with the growth rate of the number of publications in a particular field. This illustrates the fact that the relevance of characteristic timescales goes further than the traditional problems of theoretical physics.

For quantum systems, the scenario is not different: the analysis of timescales was present since the beginning, becoming more relevant with the development of computation. Characteristic timescales are important tools for the investigation of quantum complex dynamics, and the search for general procedures aimed at finding them has been undertaken for decades (see [8–10] and references therein). Some of the strategies for the calculation of characteristic timescales are the saddle point approach [11], short-time perturbative expansion [9], and the use of semiclassical methods [12–15]. As reported in Refs. [10,12,16], quantum timescales depend strongly on both the observable as the Hamiltonian and, if the system is coupled to an environment, they are also influenced by the particular form of the interaction [10,17–19]. There are plenty of ongoing works in which characteristic timescales play significant roles; some examples are focused on the decay of fidelity [20–23] and on the protection of quantum states [24–26], as well as on the generation of entangled ensembles [27].

The construction of quantum models and the analysis of their dynamics is clearly central to the understanding of the phenomena observed in the laboratory, but not only that: it also tells us what effects we can expect from the formalism. In this context, Jaynes–Cummings model [28] is paradigmatic: it has been employed for decades in the modeling of experiments [29] and, due to the complex dynamics that it produces even in the rotating wave approximation, the model is an excellent tool to understand the possibilities that quantum dynamics offer us [30–33]. Quantum nonlinear oscillators, in turn, showed to be appropriate in the description of many systems of interest [34,35], and also very useful for theoretical exploration of quantum mechanics [10,12,16,17,23,36,37]. Since the Schrödinger equation is linear, it does not present sensitivity to initial conditions, as in the case

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of classical chaotic systems [38–42]. This fact raised fundamental questions related to the expectation that classical mechanic's validity domain must be contained within quantum mechanic's validity domain. Among the proposals to address this issue, there are the decoherence program [10,16,40,42–44] and the experimental uncertainties approach [17–19,45–50]. The quartic model, which is a good approximation for the interaction of a field in a Kerr medium [51,52], can be used as a platform for quantum chaos research [36,37,53–56]. A peculiar feature of this model is useful for studying the problem of the classical limit of quantum mechanics: its dynamics becomes more classical in the Newtonian sense the lower the initial classical action is, and more classical in the Liouvillian sense when classical action increases [10,12,16–19,57]. The attention to nonlinear quantum models had a recent increase related to research in biology [58–62]. As an example, we highlight the discussion on the necessity of adopting cavity quantum electrodynamics models for the description of microtubules [61] and the debate about the effects of the thermal bath in the relevant timescales in such biological systems [58–60].

In the present contribution, we chose one quartic oscillator as a model [12,18,52,63,64] coupled to a two-level system, and seek to understand, through the analysis of characteristic times related to visibility, predictability, and concurrence, how nonlinearities affect the structure of the dynamics of entanglement. By varying the parameters related to nonlinearities, we can go from the limit where we have the Jaynes–Cummings model to the other one, where the dynamics is dominated by the effects of nonlinearities. Similar models have been studied by other authors (see [35,65] and references therein), but, to our knowledge, not in the context of entanglement timescales. Since the quartic model was performed in the laboratory, with a good agreement with the theoretical results [34], we believe that the system investigated here can be done experimentally, by means of the interaction of the field in a Kerr medium with a Rydberg atom, which can be approximately regarded as a two-level system. In Ref. [10], characteristic times computed from power series expansions are able to give relevant scales for systems composed of quartic oscillators. On the other hand, this procedure does not lead to the timescale that characterizes appropriately the decay of visibility for the Jaynes–Cummings model [66]. In the present study, we develop a method involving direct inspection of the distributions of the constants that appear in the addends that form the physical quantities investigated; using this method, we made explicit the conditions for the decay timescales to be correctly calculated by means of power series expansions. Under such conditions, the use of power series expansions leads to timescales equivalent to the ones computed through the inspection of distributions of constants. The method described here showed to be useful also for the computation of other timescales (not strictly related to decay) for the majority of cases investigated. This paper is organized as described below. In the next section, we present the model and its solution. In Section 3, we calculate the visibility, predictability and concurrence for the system under study. In Section 4, we perform the definitions of different characteristic timescales and propose ways to calculate them, always based on the inspection of distributions of constants. The investigation of such timescales for different parameters of the Hamiltonian and initial conditions is shown in Section 5. The final considerations are found in Section 6.

2. The model

Consider a quartic oscillator coupled to a two-level system according to the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2, \tag{1}$$

where

$$\hat{H}_0 = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \gamma_1\hbar\left(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right),$$
$$\hat{H}_1 = \hbar^2\lambda\left(\hat{a}^\dagger\hat{a}\right)^2, \qquad \hat{H}_2 = \gamma_2\hbar^2\left(\hat{a}^\dagger\hat{a}\hat{\sigma}_z\right), \tag{2}$$

ω, ω_0, γ_1, λ and γ_2 are real constants, $\hat{a}^\dagger$ and $\hat{a}$ are bosonic operators, and $\hat{\sigma}_+$, $\hat{\sigma}_-$ and $\hat{\sigma}_z$ are Pauli operators. The operator $\hat{H}_0$ corresponds to the Jaynes–Cummings model in the rotating wave approximation [28], which is usually employed to model a two-level atom interacting with a field mode in the vacuum. The operator $\hat{H}_1$, in turn, is proportional to the third order nonlinear susceptibility in the rotating wave approximation, and the operator $\hat{H}_2$ accounts for the dispersive atom-field interaction [34,67].

This Hamiltonian can be used to model a Rydberg atom in a Kerr medium. In the early eighties, Drummond and Walls derived a quantum model for dispersive optical bistability as a quartic oscillator [67]. This nonlinear Hamiltonian was intensively investigated by others [12,34,36,37,64,68–73]. Recently, Kirchmair and collaborators analysed the interaction of a strong field with an atom in the dispersive regime, and found the term $\sigma_z a^\dagger a$ as the most relevant [34]. This kind of interaction has been also studied by [68,70,71]. The experimental investigation of a Rydberg atom in a Kerr medium was performed by Mohapatra and collaborators who varied the regime of interaction from resonant to dispersive [71].

In what follows, $|g\rangle$ and $|e\rangle$ denote the fundamental and excited states of the two-level system, respectively, and $|n\rangle$ are Fock states. Clearly, $|g,0\rangle$ is an eigenstate of $\hat{H}$ with eigenvalue $E_0^+ = -\hbar\omega_0/2$. By observing that this Hamiltonian preserves the total number of excitations, one can calculate the remaining eigenvalues and eigenvectors, given by the expressions below, valid for $n \geq 1$:

$$\hat{H}\left|E_n^\pm\right\rangle = E_n^\pm\left|E_n^\pm\right\rangle, \tag{3}$$

where

$$E_n^\pm = \frac{f_{22n} + f_{11n} \pm \sqrt{\left(f_{22n} - f_{11n}\right)^2 + 4\left(f_{12n}\right)^2}}{2},$$
$$\left|E_n^\pm\right\rangle = \alpha_n^\pm\left|e, n-1\right\rangle + \beta_n^\pm\left|g, n\right\rangle, \tag{4}$$

with

$$\alpha_n^\pm = \frac{\pm f_{12n}}{\sqrt{\left(f_{12n}\right)^2 + \left(E_n^\pm - f_{11n}\right)^2}},$$
$$\beta_n^\pm = \frac{\pm\left(E_n^\pm - f_{11n}\right)}{\sqrt{\left(f_{12n}\right)^2 + \left(E_n^\pm - f_{11n}\right)^2}}, \tag{5}$$

and

$$f_{11n} = \left(\hbar\omega + \gamma_2\hbar^2\right)(n-1) + \frac{1}{2}\hbar\omega_0 + \hbar^2\lambda(n-1)^2,$$
$$f_{22n} = \left(\hbar\omega - \gamma_2\hbar^2\right)n - \frac{1}{2}\hbar\omega_0 + \hbar^2\lambda n^2, \qquad f_{12n} = \gamma_1\hbar\sqrt{n}. \tag{6}$$

3. Calculation of visibility, predictability and concurrence

If the state of the system is pure at the time t, it can be written in the form

$$|\Psi(t)\rangle = a_{g,0}(t)\,|g\rangle\,|0\rangle + \sum_{n=1}^{m}\left(a_{e,n-1}(t)\,|e\rangle\,|n-1\rangle + a_{g,n}(t)\,|g\rangle\,|n\rangle\right), \tag{7}$$