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Bifurcation phenomena in a semiconductor superlattice subject to a tilted magnetic field

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ABSTRACT

The paper studies instabilities of charge transport in strongly coupled semiconductor superlattices with an applied electric and a tilted magnetic field. We reveal the bifurcation phenomena, which are associated with the transitions between different regimes of charge dynamics, and also investigate effects of the temperature on these bifurcations. In addition, we find out that the development of an instability can be accompanied by a graduate change of the dominant transport mechanism.

Introduction

Semiconductor superlattices (SSLs), nanostructures formed by several alternating layers of different semiconductor materials [1–4], are subjects of a great research interest for both the fundamental and applied sciences [1,5–10]. Being biased by an applied electric field they are able to demonstrate a large number of interesting quantum-mechanical phenomena such as Wannier–Stark ladders, sequential and resonant tunneling, Bragg reflections, and Bloch oscillations. These phenomena strongly influence collective charge transport along the SSL inducing negative differential conductivity and traveling charge domains of high concentration [3]. In Ref. [11] it has been theoretically shown that two main types of charge domains can be generated in transferred electron devices with negative differential conductance, namely the pure accumulation domains and dipole domains. With this, Ref. [12] has reported the detailed experimental study indicating that the current oscillations in superlattices are most likely occurring in the pure charge accumulation mode. Recently, it has been found out that a tilted magnetic field applied to an SSL can strongly affect the electron drift velocity in this nanostructure [13,14] and, correspondingly, the dynamics of the SL in regime of charge domains propagation [15,16].

From the viewpoint of collective charge dynamics, an SSL can be considered as an active nonlinear medium, where the spatio-temporal patterns (e.g. high concentration charge domains) can be generated by a voltage applied to the contacts of the SSL [3,10]. When the applied voltage is small, spatially extended patterns of charge concentration are stationary in time. For higher voltage the stationary state becomes unstable and charge domains start to propagate along the SSL, thus generating the current oscillations. One of the typical instabilities giving birth to the current oscillations in the SSL is the supercritical Hopf bifurcation [17,18]. In this case, the current oscillations in the vicinity of the bifurcation are close to be harmonic. With further increase of the voltage, the shape and timescales both of the moving charge domains and the related current oscillations are considerably changed. In the absence of the magnetic field these modifications are rather gradual, whereas the presence of a tilted magnetic field seems to induce additional bifurcation phenomena, which are not clear at the moment [16].

In this paper we study the bifurcations induced by a tilted magnetic filed, and investigate, how the change of temperature affects the instabilities. The structure of the paper is the following. Section 1 presents the mathematical model describing the charge transport in the SSL biased by an electric and a tilted magnetic

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field. The evolution of the charge dynamics with variation of the voltage applied is discussed in Section 2. The instabilities occurring in the system and the underlying mechanism for low temperatures are considered in Section 3. Section 4 is devoted to the transport regimes and the bifurcation phenomena at high temperatures. The final remarks are given in the Conclusions.

1. Model equations

In order to describe the collective charge dynamics in the SSL we use a set of dimensionless current continuity and Poisson equations [19,20]:

$$\frac{\partial n}{\partial t} = -\beta \frac{\partial J}{\partial x}, \tag{1}$$

$$\frac{\partial F}{\partial x} = \nu(n-1). \tag{2}$$

In Eqs. (1) and (2) the dimensionless volume electron density, electric field and current density are denoted as $n(x,t)$, $F(x,t)$ and $J(x,t)$, respectively, x and t are the dimensionless space and time variables, $\beta = 3.1 \times 10^{-2}$, $\nu = 15.8$ are the dimensionless control parameters. The dimensionless quantities are related with the physical (primed) ones as:

$$x = x'/L', \quad t = t'/\tau', \quad n = n'/n'_D,$$
$$J = J'/(en'_D v'_0), \quad F = F'/F'_c, \quad F'_c = \hbar/(ed'\tau'),$$
$$\beta = v'_0\tau'/L', \quad \nu = L'en'_D/(F'_c\varepsilon_0\varepsilon_r), \tag{3}$$

where $d' = 8.3$ nm and $L' = 115.2$ nm are the period and the length of the superlattice, $e > 0$ is the magnitude of the electron charge, $\Delta' = 19.1$ meV is the miniband width, $n'_D = 3 \times 10^{22}$ m^{-3} is the n-type doping density in the SL layers, $F'_c = 3.2 \times 10^5$ V/m is the normalization value of the electric field, ε_0 and $\varepsilon_r = 12.5$ are the absolute and relative permittivities, respectively. The quantity:

$$v'_0 = \gamma \frac{\Delta' d'}{2\hbar} \frac{I_1(\Theta)}{I_0(\Theta)} \tag{4}$$

is the maximal possible value of the dimensionless drift velocity without the tilted magnetic field, where

$$\Theta = \Delta'/(2k'_B T') \tag{5}$$

characterizes the temperature T', while $I_0(x)$ and $I_1(x)$ are the modified Bessel functions of the first kind. Parameters $\gamma = [\tau'_e/(\tau'_e + \tau'_i)]^{1/2}$ and $\tau' = \gamma\tau'_i$ are determined by the scattering events. These parameters depend on the elastic τ'_e and inelastic τ'_i scattering times. In our study we use the following values: $\tau' = 250$ fs and $\gamma = 1/8.5$. The values of the physical quantities are taken from recent experimental works [14,21].

Within the drift-diffusion approximation the dimensionless current density can be written as:

$$J = n v_d(F) - D(F)\frac{\partial n}{\partial x}, \tag{6}$$

where $v_d(F)$ is the dimensionless electron drift velocity ($v_d = v'_d/v'_0$) and $D(F)$ is the diffusion coefficient [3]:

$$D(F) = v_d(F) d \frac{\exp(-\kappa F)}{1 - \exp(-\kappa F)},$$
$$\kappa = \frac{\hbar}{k'_B T'\tau'} = \frac{\hbar\Theta}{\Delta'}, \qquad d = \frac{d'}{L'}. \tag{7}$$

The diffusion coefficient (7) may be neglected when $T' \to 0$ ($\Theta \to \infty$). If there is no tilted magnetic field, the drift velocity

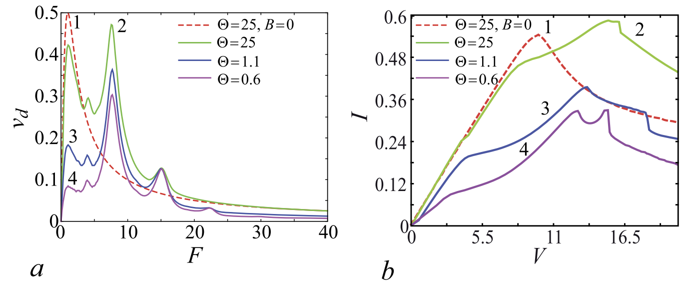

Fig. 1. The dependences $v_d(F)$ (a) and $I(V)$-characteristics (b) for the case without the tilted magnetic field (the dashed line 1) and with a tilted magnetic field (solid lines 2–4). The curves 1 and 2 correspond to the parameter value $\Theta = \Theta_1 = 25$ ($T' = 4.2$ K), curve 3 to $\Theta = \Theta_2 = 1.1$ ($T' = 100$ K), and curve 4 to $\Theta = \Theta_3 = 0.6$ ($T' = 200$ K).

$v_d(F)$ is governed by the Esaki–Tsu formula [1], which in its dimensionless form can be written as:

$$v_d(F) = \frac{F}{1 + F^2}. \tag{8}$$

In this case the dependence of the drift velocity on the electric field demonstrates only two extrema at $F_c = \pm 1$ (Esaki–Tsu peaks). In the presence of a tilted magnetic field the drift velocity $v_d(F)$ for an arbitrary temperature can be obtained numerically, e.g. using the approach described in [16]. In our calculations we apply a magnetic field $B' = 15$ T tilted with respect to the SL axis x at an angle of $\alpha = 40°$.

The calculated dependencies $v_d(F)$ for different Θ are shown in Fig. 1(a). One can see that for all temperatures the $v_d(F)$ curves exhibit multiple maxima. The first maximum observed for the lowest value of $F = F_c$ is the Esaki–Tsu peak, which is associated with the onset of the Bloch oscillations. Nonlinear interaction between the electronic Bloch oscillations along the SL and cyclotron motion in the plane of the layers induces chaotic semiclassical electron dynamics, which, depending on the ratio between the Bloch and cyclotron frequencies, either accelerate or decelerate charge transport through the SL [13,14]. As a consequence we observe other maxima on the dependencies $v_d(F)$ corresponding to the Bloch-cyclotron resonances, which occur due to the resonant acceleration of the electrons whenever the ratio of the Bloch and cyclotron frequencies equals $r = 0.5, 1, 2, \ldots$ (Bloch-cyclotron resonances) [13]. Thus, at the presence of a tilted magnetic field there are two major transport mechanisms, namely the conventional Esaki–Tsu transport [1] and the Bloch-cyclotron resonances, when the Bloch and cyclotron frequencies are commensurate, the electrons exhibit a unique type of quantum chaos, which does not obey the Kolmogorov–Arnold–Moser theory [22]. This type of chaos is characterized by the formation of intricate web-like structures, known in the literature as "stochastic webs" [22,23], which extend throughout the phase space of the miniband electrons. The appearance of these webs abruptly delocalizes electrons in real space, thus significantly increasing their drift velocity due to nonlinear interaction between the Bloch oscillations and cyclotron motion [13,15].

Remarkably, as the temperature increases (Θ decreases), the Esaki–Tsu peak dramatically weakens, whereas the resonant peaks become more prominent [16]. Moreover, new resonant peaks arise from the background with the drop of Θ (compare curves 2, 3 and 4).

The dimensionless bias (constant) voltage $V = V'/(F'_c L')$ applied to the SSL creates a global constraint:

$$V = U + \int_0^1 F\,dx, \tag{9}$$