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Spatio-temporal autocorrelation measures for nonstationary series: A new temporally detrended spatio-temporal Moran's index

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ABSTRACT

In order to measure the spatio-temporal autocorrelation's degree for spatio-temporal nonstationary series, the new temporally detrended global and local spatio-temporal Moran's indexes (TDGSTI and TDLSTI) are proposed. The implementation of the new Moran's indexes is illustrated through artificial and real examples. Analyses of the influencing factors on TDGSTI are performed. A statistical test of TDGSTI is taken. The Moran's scatter plot, which discloses the spatio-temporal cluster pattern's characteristics and pattern's change, is extended. TDGSTI is found to reveal the autocorrelation level of spatio-temporal objects. For a positive TDGSTI, the higher the TDGSTI, the higher the autocorrelation level, and vice versa. TDGSTI is closely related to time-scale s, time-lag h and spatio-temporal weight matrix. For $s \gg h$, TDGSTI is significant, while for $s \sim h$ and $s < h$, TDGSTI is insignificant. TDGSTI has clear potential to test the spatio-temporal autocorrelation's degree for spatio-temporal nonstationary series in other research fields.

1. Introduction

The spatio-temporal autocorrelation analysis is an effective method for gaining insight into spatio-temporal structures and patterns. Sophisticated models and methods have been proposed and developed to account for autocorrelation in spatial [1,2] and temporal [3] data, and a profusion of researches have been focusing on both domains. In traditional geostatistics, many methods and indexes are used to test spatial dependency, such as Moran's I [4], Geary's C [5] and Getis-Ord general G [6]. Moran's I is of particular importance, and is often applied to evaluate autocorrelation's level among the attributive values of the spatial objects in a variety of research fields.

The concept of spatio-temporal autocorrelation was first proposed by Cliff [7]. And then, spatio-temporal autocorrelation function and spatio-temporal partial correlation function were proposed to measure spatio-temporal dependency quantitatively [8]. Wang [9] and López [10] extended the univariate Moran's I, and presented spatio-temporal Moran's I. Chen also showed a spatio-temporal Moran's I using the improved analytical method based on the traditional Moran's I [11]. By comparison, it is found that the global and local Moran's indexes, presented by Wang, López and Chen, are actually consistent with each other. Further analysis of the spatio-temporal Moran's I shows that the attributive variable's time series in a spatial location is implicitly assumed to be stationary by Chen [11].

However, the time series in a spatial location is commonly nonstationary in the real world. This time series in the presence of nonstationarity violates the implicit assumption: stationarity. Thereby, a spatio-temporal autocorrelation between nonstationary series might be spurious when considered from the viewpoint of the spatio-temporal Moran's I proposed by Chen [11].

Fortunately, a method, detrended fluctuation analysis (DFA), has been proposed to analyze power-law correlations of nonstationary time series [12,13]. By detrending local trends, DFA can ensure that the results are not affected by trends (including linear, quadratic, and even higher order trends and periodic trends) [14]. Afterwards, a detrended cross-correlation analysis (DCCA) is developed to test the cross-correlation between nonstationary time series based on a detrended covariance [15–17]. DCCA is a generalization of DFA method. Earlier studies have confirmed that DCCA and DFA can uncover more hidden correlation information than other analyses, leading to their acceptance and application in diverse fields [18–25].

In this paper, the spatio-temporal series is confined in the content, and is designated that with time, the attributive value of a single variable in a specific spatial object constitutes one time se-

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ries, and multiple time series in different spatial objects are combined into the spatio-temporal series. The spatio-temporal series is, of course, composed of spatio-temporal objects. We assume that the time series is nonstationary, and spatial objects are stationary for simplicity. Thus, in the case of a single spatial object, spatio-temporal autocorrelation thus reduces to temporal autocorrelation only.

With the goal of reducing the effect of time series' nonstationarity on a spatio-temporal autocorrelation, temporally detrended global and local spatio-temporal Moran's indexes (TDGSTI and TDLSTI) are proposed to quantify the autocorrelation's level among spatio-temporal objects for possibly nonstationary and long-range dependent series at different time scales, based on spatio-temporal dependency and a DFA regression framework [26,27]. In order to more deeply and effectively understand the mechanism, and to apply TDGSTI and TDLSTI to solve practical, real-world problems, we identify and quantify the spatio-temporal autocorrelation of both an artificial series and real-precipitation series in north Jiangsu, China. As a result, it is found that TDGSTI is capable of measuring spatio-temporal autocorrelation for potential nonstationary time series and long-range correlation data at different time scales, and the extended Moran's scatter plot can uncover the spatio-temporal cluster patterns.

2. Methods

2.1. Temporally detrended global spatio-temporal Moran's index

Spatial autocorrelation is a fundamental characteristic between an attributive value of a variable in a spatial object and the values in adjacent spatial objects [1]. If an attributive value in a spatial object is independent of the values of the neighboring objects, spatial process is stochastic [30]. In this case, there is no spatial autocorrelation among the spatial objects. An emphasis of spatial autocorrelation is that each variable has the same nature. The traditionally global Moran's index [1,4] provides a formal indication to measure the linear-association degree between the attributive values of all the spatial objects.

With the traditional Moran's index [1], a vector with its dimension of n is supposed as follows: $x = [x_1, x_2, \ldots, x_i, \ldots, x_n]$ $(1 \le i \le n)$, where x_i is an attributive value in the ith spatial object. The traditionally global Moran's index can be written in matrix notation as $I = ZWZ^T$ [1], where the parameter I is the traditionally global Moran's index, Z is a vector with its dimension of n about the variable $x : [Z_i = (x_i - \bar{x})/\sigma]$ $(1 \le i \le n)$, $\bar{x} = \sum_{i=1}^{n} x_i/n$ denotes mean, σ is a standard deviation of the variable x, and W is a spatial weight matrix, defining existing interaction between spatial objects.

The geometric meaning of the traditionally global Moran's index is interpreted to be a linear regression coefficient between WZ^T and Z^T [30–32], namely, the corresponding linear regression model is written as Eq. (1):

$$WZ^T = IZ^T + \alpha + \varepsilon \tag{1}$$

where α is a constant term, ε is an error term (residual) with spatial variation, and WZ^T is a spatial-lagged vector to determine the states of the adjacent spatial objects.

We suppose that spatio-temporal process is autocorrelated if an attributive value in a spatial-temporal object is dependent on the values of the neighboring objects.

Thus, for the purpose to measure the spatio-temporal autocorrelation, we suppose that there are N spatial objects, the attributive value of the variable in each spatial object varies with time to make up a time series with in the presence of nonstationarity and long-range correlation, and the length of this time series is T. Each variable in N spatial objects has the same nature. Thereby, N time series in N spatial objects are linked to make up a spatio-temporal original series in the presence of nonstationarity and long-range correlation with its length of NT. For this original series, it is assumed that an analogous model as Eq. (2) is adopted to measure spatio-temporal dependency:

$$W^{st}X^T = I_{st}X^T + \alpha + \varepsilon \tag{2}$$

where the parameter I_{st} is a spatio-temporal Moran's index, indicating the linear-association degree between the attributive values of spatio-temporal objects; X is a vector for the spatio-temporal original series with its dimension of NT; W^{st} is a spatio-temporal weight matrix with its row standardization (a weight matrix element in each row divided by its row sum), defining the interactions between a specific spatio-temporal object and adjacent spatio-temporal ones; α is also a constant term, and ε is also an error term with spatio-temporal variation. $Y^T = W^{st}X^T$ is denoted as a spatio-temporal-lagged vector with its dimension of NT, representing a weighted average of the adjacent spatio-temporal objects. Hence, Eq. (2) is simplified into $Y^T = I_{st}X^T + \alpha + \varepsilon$.

Clearly, the estimation of parameter I_{st} is crucial to empirical studies across disciplines [26]. However, the estimation of the parameter I_{st} does not satisfy the assumptions of steady variance and covariance, which are required by the standard regression analysis using ordinary least squares method (OLS), because the spatio-temporal original series is nonstationary.

Recently, a regression framework for possibly nonstationary and long-range correlation series was presented [26,27] on basis of DFA [12,13]. Based on Kristoufek's study work [26], a new temporally detrended spatio-temporal Moran's analysis is proposed, and a temporally detrended spatio-temporal Moran's index is defined to quantify the autocorrelation's level among the spatio-temporal objects. The calculation procedure of a temporally detrended global spatio-temporal Moran's index can be summarized in the following steps:

Step 1. Construct spatio-temporal series.

The spatio-temporal-original series, $T_x = \{x(1,1), x(1,2), \ldots, x(1,T), x(2,1), x(2,2), \ldots, x(2,T), \ldots, x(p,i), \ldots, x(N,T)\} = \{X(1), X(2), \ldots, X(u = (p-1)T + i), \ldots, X(NT)\}$ $(1 \le p \le N,\ 1 \le i \le T,\ 1 \le u \le NT)$, is combined by N time series with each length of T [11]. The length of this original series T_x is NT. Note that $x(p,i) = X(u = (p-1)T + i)$ is the ith timepoint in the pth spatial object for T_x.

A spatio-temporal-lagged vector Y, $Y^T(h) = W^{st}(h)X^T$, is converted into a spatio-temporal-lagged series $T_y = \{y(1,1,h), y(1,2,h), \ldots, y(1,T,h), y(2,1,h), y(2,2,h), \ldots, y(2,T,h), \ldots, y(q,j,h), \ldots, y(N,T,h)\} = \{Y(1,h), Y(2,h), \ldots, Y(v = (q-1)T + j, h), \ldots, Y(NT,h)\}$ $(1 \le q \le N,\ 1 \le j \le T,\ 1 \le v \le NT)$, where $W^{st}(h)$ is a spatio-temporal weight matrix with NT rows and NT columns, and h is time lag for the temporal weight function for simplicity [8].

Step 2. Calculate detrended covariance and variance between spatio-temporal series: T_x and T_y.

First, two integrated series $R_k = \sum_{u=1}^{k}(X(u) - \overline{X})$ and $R'_k(h) = \sum_{v=1}^{k}(Y(v,h) - \overline{Y}(h))$ $(1 \le k \le NT)$ are computed, where $\overline{X}$ and $\overline{Y}(h)$ are the averaging over the two full spatio-temporal series with each length of NT. Next, the entire time series are divided into $NT - s$ overlapping boxes, each containing $s + 1$ values. Then, the covariance $f^2_{xy}(s,h,u)$ of the residuals in a box of size s (time scale) that starts at u and ends at $u + s$ is calculated as Eq. (3):