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# Early detection of rogue waves by the wavelet transforms

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ABSTRACT

We discuss the possible advantages of using the wavelet transform over the Fourier transform for the early detection of rogue waves. We show that the triangular wavelet spectra of the rogue waves can be detected at early stages of the development of rogue waves in a chaotic wave field. Compared to the Fourier spectra, the wavelet spectra are capable of detecting not only the emergence of a rogue wave but also its possible spatial (or temporal) location. Due to this fact, wavelet transform is also capable of predicting the characteristic distances between successive rogue waves. Therefore multiple simultaneous breaking of the successive rogue waves on ships or on the offshore structures can be predicted and avoided by smart designs and operations.



## 1. Introduction

Rogue waves, waves with a height more than 2–2.2 times the significant wave height in the wave field [1], present a danger to life and their result can be catastrophic and costly. The early detection of rogue waves in the chaotic ocean is a must for the early-warning systems that ensure the safety of the marine travel and the offshore structures in stormy conditions. However the early detection of rogue waves is an extremely hard problem. First of all rogue waves appear in stormy conditions therefore accurate prediction of weather conditions is a must. Secondly rogue waves appear and disappear in a length (time) scale that is on the order of their width [2]. Due to the rapidly changing nature of the rogue waves, the reliability of the forecast of the rogue waves at the current stage is not high and it is hard to expect that it will become more reliable in the near future [2].

This vital problem has been disregarded for a long time and has only been studied for almost a decade [2,3]. The proposed approach in [2] is to continuously measure the part of the whole surface spectrum in real time and use the triangular Fourier spectra of the growing rogue waves in early stages of their development in a chaotic wave field before the peak appears [2].

In this paper, we show that using the wavelet transforms for the early detection of rogue waves over or with the Fourier transforms is more advantageous than using the Fourier transforms solely. With this motivation, we use a simple model similar to the one discussed in [2]. In our model the complications due to the two-dimensional effects, finite water depth, higher-order dispersion and higher-order nonlinearity and other factors that exist in reality are ignored [2]. We numerically generate a chaotic wave field and analyze the wavelet spectra before and after the rogue wave formation. We show that the wavelet spectra exhibit V-shaped high energy regions when high waves are formed. In the case of a rogue wave occurrence we see that at very low scales of the wavelet transform there is energy. This property can be used for the early detection of the rogue waves. Additionally using the numerical model we show that wavelet analysis is also capable of finding the possible location of a rogue wave thus characteristic distances between successive rogue waves (such as the famous "three-sisters") can be predicted as well. Therefore wavelet transforms are more advantageous over the Fourier transforms for the early detection of rogue waves when they are used in a model or in the measurements. Using this result, multiple simultaneous breaking of the successive rogue waves on ships or on offshore structures can be predicted. So by means of smart designs and operations, the safety of the ocean travel and offshore operations can be enhanced.

## 2. Generation of the rogue waves in a chaotic ocean wave field

### 2.1. Review of the nonlinear Schrödinger equation and Peregrine soliton

Dynamics of weakly nonlinear deep water ocean waves are described by the nonlinear Schrödinger equation (NLSE) [4,5]. One of the most common forms of the NLSE is given by

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$$i\psi_t + \frac{1}{2}\psi_{xx} + |\psi|^2 \psi = 0 \quad (1)$$

where $x, t$ are the spatial and temporal variables, $i$ denotes the imaginary number and $\psi$ is complex amplitude. This notation is mainly used in ocean wave theory whereas $t$ and $x$ axes are switched in fiber optic studies. NLSE is also widely used in other branches of the applied sciences and engineering to describe various phenomena including but not limited to pulse propagation in optical fibers and quantum state of a physical system. Integrability of the NLSE is studied extensively within last forty years and some exact solutions of the NLSE are derived. Some rational soliton solutions of the NLSE are derived as well. One of the most early forms of the rational soliton solution of the NLSE is the Peregrine soliton [6]. It is given by

$$\psi_1 = \left[1 - 4\frac{1+2it}{1+4x^2+4t^2}\right]\exp[it] \quad (2)$$

where $t$ is the time and $x$ is the space parameter. It is shown that Peregrine soliton is a first order rational soliton solution of the NLSE and the higher order rational solutions of the NLSE and a hierarchy of obtaining those rational solutions based on Darboux transformations are given in [7]. Details of the Darboux transformations can be seen in [8]. It has been confirmed that the hydrodynamic rogue waves are in the Peregrine soliton form [9]. Additionally, throughout many simulations it has been confirmed that rogue waves obtained by numerical techniques which solve the NLSE are in the forms of these first (Peregrine) and higher order rational solutions of the NLSE [2,7,10].

One of the major problems with the early detection of the rogue waves is their rapidly changing nature. In the literature rogue waves are described as the waves that appear from nowhere and disappear without a trace. A recent analysis given in [11] which employs the Grassberger–Procaccia nonlinear time series algorithm opposes this description and suggests that early warning times for rogue waves can be enhanced. However as explained in [11], at best one may expect to predict an ocean rogue wave a few ten seconds before impact with the current understanding of the rogue wave dynamics. In order to illustrate the rapidly changing nature of the rogue waves in the physical space we give the following numerical example. Consider the Peregrine soliton given in (2) and the scale, phase and Galilean transformations [12]

$$\psi(x,t) \to B\psi(Bx, B^2 t), \quad B \in \Re^+ \quad (3)$$

$$\psi(x,t) \to \exp[ic]\psi(x,t), \quad c \in \Re \quad (4)$$

$$\psi(x,t) \to \psi(x - Vt, t)\exp[iVx - iV^2 t/2], \quad V \in \Re \quad (5)$$

We obtain a progressive Peregrine soliton by using (2) and (5) together and we set $V = 25$ m/s. Initial profile at $t = 0$ s and the final profile obtained at $t = 2$ s of time stepping is compared in Fig. 1. As depicted in Fig. 1, within 2 s of time the Peregrine soliton almost vanishes.

The Fourier transform of the Peregrine soliton can analytically be calculated as

$$F(k,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \psi(t,x)e^{ikx}dx \quad (6)$$

which leads to

$$F(k,t) = \sqrt{2\pi}\left[\frac{1+2it}{\sqrt{1+4t^2}}\exp\left(-\frac{|k|}{\sqrt{2}}\sqrt{1+4t^2}\right) - \delta(k)\right] \times \exp[it] \quad (7)$$

where $k$ is the wavenumber parameter and $\delta$ is the Dirac-delta function [2]. In Fig. 1, we also present the Fourier transforms of the Peregrine solitons in the initial and final times. As discussed in [2], the triangular shape of the Fourier spectra can be used for the early detection of the rogue waves. This is also validated for a chaotic wave field in [2].

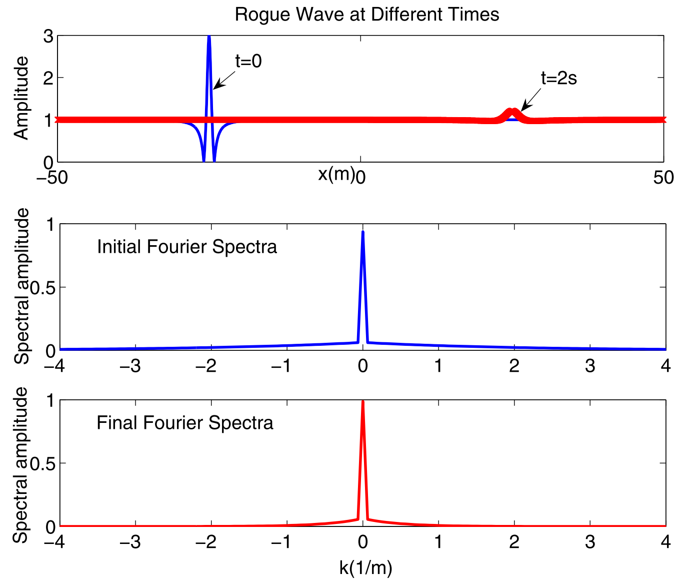


**Fig. 1.** Temporal evolution of a rogue wave and its Fourier transform.

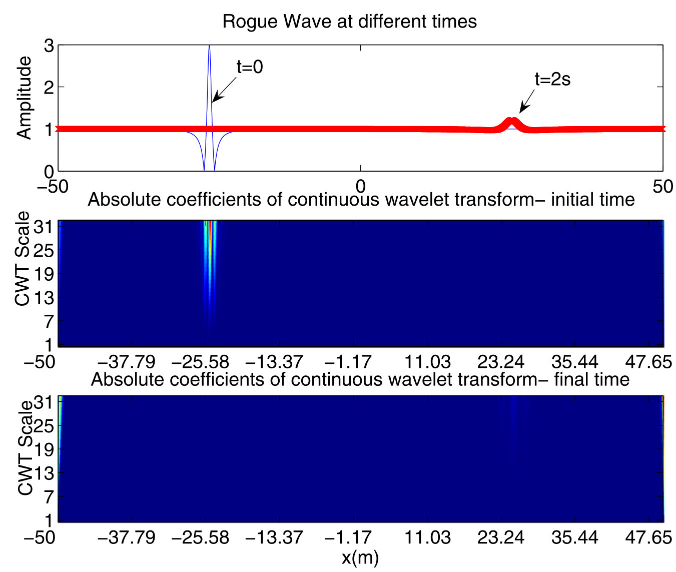


**Fig. 2.** Temporal evolution of a rogue wave and its wavelet transform.

We propose that using the wavelet transforms in a similar fashion can be more advantageous for the early detection of rogue waves. It is known that wavelet transforms preserve the temporal variations when they are used for calculating the spectra of a time series [13–15]. Therefore they are capable of finding the spatial (temporal) location of the changes in the wavenumber (frequency) [13–15]. Therefore wavelet transforms can detect not only if a rogue wave will develop but also where it will develop in the spatial (or temporal) domain.

There are many different wavelets such as symlet, Daubechies, coiflet, biorthogonal, Meyer, Haar, etc. just to name a few. Depending on the mother wavelet function it may or may not be possible to calculate the wavelet transform of the Peregrine soliton analytically. For illustrative purposes we only present numerical results with symlet wavelet of order 2. We show the wavelet transform of the Peregrine rogue wave propagating on the still water in Fig. 2.