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Discussion

Enhancement of synchronization in inter–intra-connected neuronal networks

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ABSTRACT

We study the enhancement of neural synchrony in a network of electrically coupled Hindmarsh Rose (HR) neurons. The behavior of the network under control by an external environment modeled by the Fitzhugh Nagumo (FN) is analyzed. Biologically, such a control system could mimic the modification of normal neuronal dynamics due to drugs or other chemical substances. We show that the environment could have as effect the suppression of chaos, enhancement of synchrony and favor interesting properties such as sub-threshold membrane oscillations, and oscillation death for relatively strong local coupling. Interestingly, we find that the electrical coupling between each two coupled HR and FN is less important to synchronization than the local coupling between the HR and the FN neurons. In other words, local interactions are found to play a stronger role in synchronization than long-range (global) interactions.

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1. Introduction

A common approach to study synchronization is to consider a network in which the neurons are coupled either chemically or electrically. Synchronization is then analyzed by varying a single parameter that determines the coupling strength between the neurons. The simplistic nature of this architecture limits the level of control over the system and has motivated studies of the combined effect of chemical and electrical synapses on synchronization [1–12]. In this case, two parameters are required to model the network topology, allowing more flexibility in the control of synchronization properties of the system.

In the present work, we consider an even more flexible neural network architecture in which each unit is a 3 dimensional (3D) system, which is dynamically coupled to a 2D control. We represent the 3D system with the HR neuron [13,14], whose dynamic is given by Eq. (1).

$$\dot{x} = y + ax^2 - x^3 - z + I_e$$

$$\dot{y} = 1 - bx^2 - y$$

$$\dot{z} = r[s(x - x_o) - z] \qquad (1)$$

where $x(t)$ represents the membrane potential, which is written in dimensionless units. The spiking variables $y(t)$ and the bursting variable $z(t)$ describe the rates of ion transport through fast and slow ion channels respectively. The variable a controls the spiking activity of the HR neuron and allows the switching between bursting and spiking. r controls the speed of variation of the slow (bursting) variable and s is a recovery parameter. Following the literatures, values adopted here are $a = 1$, $b = 5$, $r = 0.006$, $s = 4$. The external input current I_e mimics a membrane input excitation current. A minimum initial excitation current of $I_e \approx 1.37$ is required to set the HR into an active state. In this work, we use $I_e = 3.2$, a value at which the HR is chaotic. The 2D control system is modeled by the spiking Fitzhugh Nagumo (FN) neuron [15], whose dynamics is given by Eq. (2).

$$\dot{u} = u - u^3 - v \qquad (2)$$

$$\dot{v} = u + a_o - b_o v,$$

in which u is the membrane potential and v is the recovery variable representing the force that restores the resting state of the FN system. The parameters are chosen as $a_o = 0.05$ and $b_o = 0.06$, to allow for rapid spiking.

The dynamical properties of the FN and HR neurons are well known and several studies on the synchronization properties of the latter have been reported. However, it is uncertain how a local coupling of these two systems will affect synchronization when a network architecture is considered. Our goal is to study the effect of such local couplings on synchronization. The rest of this work is organized as follows: In Section 2, we analyze the synchronization behavior of a single system unit, consisting of a HR neuron coupled to an FN neuron. An analysis of the bifurcation route of the oscillator is presented. Section 3 deals with the stability of the synchronized system in a network architecture. We determine the contributions of local coupling (interactions *within* a single network unit) and global coupling (interactions *between* network units) to the stability of the synchronized network. Our conclusions and remarks appear in Section 4.

2. The coupled HR and FN system

The HR and FN models have independently been widely used in computational studies of neural networks. These systems are well known for exhibiting a rich variety of neuronal behaviors, including bursting and spiking. In the absence of electrical synapses, we represent the dynamics of a single unit of coupled HR and FN by

$$\dot{x} = y + ax^2 - x^3 - z + I_e + \epsilon_1 u$$
$$\dot{y} = 1 - bx^2 - y$$
$$\dot{z} = r[s(x - x_o) - z] \tag{3}$$
$$\dot{u} = u - u^3 - v + \epsilon_2 x$$
$$\dot{v} = u + a_o - b_o v.$$

In Eq. (3), the parameters ϵ_1 and ϵ_2 are used to control the local coupling within a single network unit. For the rest of our analysis and without loss of generality, we will assume $\epsilon_1 = \epsilon_2 = \epsilon$. In engineering, a control could be represented by a mechanical system [16,17]. In biological neurons, a control could model the modification of normal neuronal dynamics, due to the action of drugs or other chemical substances introduced in the system. We note that communication between neurons entirely depends on the generation and propagation of the action potential. This is a chemical process that involves a change in ion concentration across the cell membrane. It is therefore not controversial to state that a chemical substance can trigger electrical synapses. The main difference between electrical and chemical synapses resides on the range of interaction. The former are *short range* while the latter are *long range*.

The time series for the coupled systems for small values of ϵ are plotted in Fig. 1. When the control variable is set to zero Fig. 1(a) and (b), we observe the normal spiking and bursting behaviors for the FN and the HR respectively. That is, the two neurons operate on different duty cycles (fraction of time during which the neuron is active). For relatively small ϵ (weak coupling), we observe that bursting is damped in the HR while the duty cycle of the FN is reduced (Fig. 1(c) to (f)). This suggests that the two neurons exchange their properties towards the achievement of an equilibrium state. Interestingly, we find that the HR neuron is now active even for $I_e = 0$, indicating the emergence of a self-regenerative excitation property. This implies that the system is less sensible to initial conditions and has a higher propensity to achieve synchronization under the action of the control system (FN).

On the other hand, Fig. 2 shows time series of the HR and FN for large values of ϵ (strong coupling). A large frequency mismatch (Fig. 2(a) and (b)) is observed, and the time series are patterns of spike-trains interrupted by periods of sub threshold membrane oscillations. This behavior has also been observed in the ION [18].

Figs. 2(c) and (d) show wave pulses, suggesting a rhythmic interaction between the neurons and enhancement of synchronization. Finally, Figs. 2(e) and (f) ($\epsilon = 8.7$) reveal complete inhibition for the two neurons characterized by persistent sub-membrane oscillations. This suggests that any incoming stimulus has only a hyper polarizing (i.e. making the membrane potential to be more negative) effect on the receiving neuron. This phenomenon is known as oscillation death.

2.1. Fixed points and their stability

In this section, the analysis of fixed point and their stability is performed. The fixed points are obtained by setting $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = \frac{du}{dt} = \frac{dv}{dt} = 0$ in equation (3). The location of the fixed points $(x_e, y_e, z_e, u_e, v_e)$ are determined by solving the coupled system:

$$\begin{cases} u_e^3 + (\frac{1}{b_0} - 1)u_e + \frac{a_0}{b_0} - \epsilon x_e = 0 \\ x_e^3 + (b - a)x_e^2 + sx_e - sx_0 - 1 - I - \epsilon u_e = 0. \end{cases} \tag{4}$$

System (4) is a set of cubic polynomials in u_e and x_e respectively, with discriminants given by:

$$\begin{cases} \Delta_{u_e} = 4\left(\frac{1}{b_0} - 1\right)^3 + 27\left(\frac{a_0}{b_0} - \epsilon x_e\right)^2 \\ \Delta_{x_e} = 4(b - a)^3(-sx_0 - 1 - I - \epsilon u_e) \\ \qquad + 27(-sx_0 - 1 - I - \epsilon u_e)^2 \\ \qquad - 54(b - a)(-sx_0 - 1 - I - \epsilon u_e) \\ \qquad - 9(b - a)^2 + 108. \end{cases} \tag{5}$$

Hence, each can admit 3 equilibrium points when $\Delta_{u_e} < 0$ (resp. $\Delta_{x_e} < 0$), 2 equilibrium points when $\Delta_{u_e} = 0$ ($\Delta_{x_e} = 0$) and a unique equilibrium point when $\Delta_{u_e} > 0$ ($\Delta_{x_e} > 0$).

We note that equilibria of dynamical systems are not always stable. Since stable and unstable equilibria play quite different roles in the dynamics of a system, we find it useful to classify the equilibrium points $(x_e, y_e, z_e, u_e, v_e)$ for our system based on their stability. Therefore, we study the stability of $(x_e, y_e, z_e, u_e, v_e)$ by analyzing the eigenvalues of the characteristic equation of the jacobian matrix given by (6), obtained by linear approximation of (3).

$$J_0 = \begin{bmatrix} 2ax_e - 3x_e^2 & 1 & -1 & \epsilon & 0 \\ -2bx_e & -1 & 0 & 0 & 0 \\ rs & 0 & -r & 0 & 0 \\ \epsilon & 0 & 0 & 1 - 3u_e^2 & -1 \\ 0 & 0 & 0 & 1 & -b_0 \end{bmatrix} \tag{6}$$

It is easy to see from properties of matrix algebra that the eigenvalues $\lambda_1 = -1$, $\lambda_2 = -r$ and $\lambda_3 = -b_0$ have negative real parts, leaving the remaining reduced matrix

$$J(x_e, u_e) = \begin{bmatrix} -3x_e^2 - 2(a - b)x_e - s & \epsilon \\ \epsilon & 1 - 3u_e^2 - \frac{1}{b_0} \end{bmatrix} \tag{7}$$

The type of equilibrium point can be determined by the signs of the trace $Tr[J(x_e, u_e)] = [-3(x_e^2 + u_e^2) - 2(a - b)x_e + (1 - s - \frac{1}{b_0})]$ and the determinant $Det[J(x_e, u_e)] = [(-3x_e^2 - 2(a - b)x_e - s)(1 - 3u_e^2 - \frac{1}{b_0}) - \epsilon^2]$ [19]. The eigenvalues $\lambda_{4/5}$ of the jacobian $J(x_e, u_e)$ are given by

$$\lambda_{4/5} = \frac{1}{2}\{Tr[J(x_e, u_e)]$$
$$\pm \sqrt{\{Tr[J(x_e, u_e)]\}^2 - 4Det[J(x_e, u_e)]}\} \tag{8}$$

Table 1 below gives a summary of possible types of equilibria according to the values of x_e, u_e, Tr_-, Tr_+, Det_- and Det_+. Tr_- and Tr_+ are respectively the negative and the positive zeroes of