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Dynamic scaling and large scale effects in turbulence in compressible stratified fluid

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We consider the propagation of sound in a turbulent fluid which is confined between two horizontal parallel plates, maintained at different temperatures. In the homogeneous fluid, Staroselsky et al. had predicted a divergent sound speed at large length scales. Here we find a divergent sound speed and a vanishing expansion coefficient at large length scales. Dispersion relation and the question of scale invariance at large distance scales lead to these results.

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1. Introduction

Theoretical studies of fully developed turbulence, particularly in the physical sciences, have dealt with structure factor primarily in the incompressible limit. The most well known result here is the Kolmogorov $-5/3$ law for the energy spectrum $E(k)$ [defined as $E = \int E(k)dk$, where E is the total kinetic energy per unit mass and k is the wave-number], which asserts that in the inertial range (the intermediate zone far removed from the large energy injecting length scales and the small energy dissipating length scales), $E(k) \sim k^{-5/3}$ [1,2]. For weakly compressible fluids the first significant result was that of Staroselsky et al. [3], who made the remarkable observation that in such fluids the turbulent fluctuations will cause the sound speed to increase at large length scales following a $k^{-1/3}$ law (to actually see the $k^{-1/3}$ behavior one would need to go to very small wave-numbers). Since there is a large background sound velocity, the turbulent contribution ($k^{-1/3}$) would have to dominate it to be clearly seen but there should be a enhancement which should be easier to detect as was indicated in the preliminary numerical work in Ref. [3]. Subsequent work [4] dealt with the interplay of the propagating sound and the eddy viscosity (scale dependent viscosity due to turbulent fluctuation [5]) and showed that it would lead to a frequency dependence of the eddy viscosity and give rise to the scale dependent sound velocity as a consequence of dynamic scaling [6]. It was noted that the frequency dependent eddy viscosity would show up in the shape of the frequency spectrum in a light scattering study of a weakly compressible turbulent fluid. It should be borne in mind that the scaling in this context is, as in all cases of fully developed turbulence, applicable to the range of k-values such that $k_L \ll k \ll k_D$, where k_L is a small wave-number corresponding to the longer wavelengths where the external force operates and pumps in energy and k_D is a large wave-number corresponding to microscales where viscosity dissipates the energy. In this sense, the scaling in turbulence is always of restricted validity.

In the last few years attention has been devoted to turbulence in stratified fluids with an imposed temperature gradient causing the density variation. What has been a major point of discussion is the issue of whether the energy spectrum will be driven by the kinetic energy flux and lead to the Kolmogorov $-5/3$ law [1] or will it be dominated by the thermal flux and lead to $E(k) \sim k^{-11/5}$ as predicted by Bolgiano [7] and independently by Obukhov [8]. The heating from below scenario remains controversial inspite of a lot of experimental [9,10] and numerical efforts [11–15]. For the stable stratification it seems that there will be a crossover [16,17] from one spectrum to the other depending on the Richardson number $R_i = \frac{\alpha(\Delta T)gd}{v_s^2}$, where ΔT is the temperature difference between two horizontal plates separated by a distance d between which the fluid is confined. The thermal expansion coefficient of the fluid is α and v_s is the root mean square velocity.

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In this work, we would like to explore the propagation of sound waves in a turbulent stratified fluid supporting a temperature gradient. We consider a scaling theory for turbulent fluctuations and thus base our arguments solely on scale invariance. This is scaling in the extended sense (dynamic scaling which incorporates transport phenomena [18]) which has successfully handled sound propagation in critical fluids [19]. Our primary result is that in addition to a long wavelength increase in the sound velocity, there will be a decrease in the expansion coefficient at long wavelengths leading to an asymptotically vanishing expansion coefficient. Of course it does not actually vanish because of the restriction on the inertial range.

2. Dispersion relation

We begin by finding the dispersion relation for sound propagation close to the steady state in a stratified fluid which is kept between two large horizontal heat-conducting parallel plates located at $z = 0$ and $z = d$. The bottom plate is maintained at a temperature T_1 and the top at T_2 with $\Delta T = T_1 - T_2$, where ΔT can be positive or negative. The stationary fluid state profile is governed by velocity $\vec{u} = 0$, temperature T_s, density ρ_s, and pressure P_s.

$$T_s(z) = T_1 - \frac{\Delta T}{d} z \tag{1a}$$

$$\frac{\rho_s(z) - \overline{\rho}}{\overline{\rho}} = \alpha\left[\overline{T} - T_s(z)\right] \tag{1b}$$

$$\frac{\partial P_s}{\partial z} = -\rho_s g \tag{1c}$$

where we have assumed that the depth of the fluid layer is small enough to be able to talk about a mean density $\overline{\rho}$. The hydrodynamic equations in the Eulerian framework read

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\frac{\vec{\nabla} P}{\rho} + \vec{g} + \nu\nabla^2\vec{u}, \tag{2a}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0, \tag{2b}$$

$$\frac{\partial T}{\partial t} + (\vec{u}\cdot\vec{\nabla})T = \lambda\nabla^2 T. \tag{2c}$$

The acceleration due to gravity is $\vec{g}$, ν is the kinematic viscosity and λ is the thermal diffusivity. The stationary state profiles are as shown in Eqs. (1a) and (1b). We want to discuss the linearization around that profile to study the propagation of sound. The first thing we need to be careful about as pointed out by Clarke and Carswell [20] is that the perturbation around the stationary state should be taken to be Lagrangian. We will consider fluctuations ΔP, $\Delta\rho$ and $\Delta\vec{u}$ around the steady state in the Lagrangian picture (i.e., these are local changes only). So that we can write

$$\Delta P = {c_s}^2 \Delta\rho \tag{3}$$

where c_s is the velocity of sound. However to use Eqs. (2a)–(2c) we need the fluctuations in the Eulerian picture $(\delta P, \delta\rho, \delta\vec{u})$ where a change may be brought about due to displacement. For a spatially uniform stationary state the two pictures are equivalent but for a stratified fluid there will be a difference. If $\vec{\xi}$ is a local displacement, for any quantity $X(\vec{r}, t)$, we have

$$\delta X(\vec{r}, t) = \Delta X(\vec{r}, t) - (\vec{\xi}\cdot\vec{\nabla})X_s(\vec{r}, t) \tag{4}$$

where X_s is the stationary value. Clearly, for the velocity field

$$\delta\vec{u} = \Delta\vec{u} \tag{5}$$

and we will denote the fluctuation henceforth by $\vec{u}(\vec{r}, t)$, since the steady state corresponds to zero velocity. For the others,

$$\delta P = \Delta P + \xi_z\rho_s g \tag{6a}$$

$$\delta\rho = \Delta\rho - \overline{\rho}\alpha\frac{\Delta T}{d}\xi_z \tag{6b}$$

$$\delta T = \Delta T + \xi_z\frac{\Delta T}{d} \tag{6c}$$

Linearizing Eq. (2b) and using $\beta = \Delta T/d$,

$$\frac{\partial\Delta\rho}{\partial t} - \overline{\rho}\alpha\beta\frac{\partial\xi_z}{\partial t} + \rho_s\vec{\nabla}\cdot\vec{u} + \overline{\rho}\alpha\beta u_z = 0 \tag{7}$$

Since $u_z = \frac{\partial\xi_z}{\partial t}$, we get

$$\frac{\partial\Delta\rho}{\partial t} + \rho_s(\vec{\nabla}\cdot\vec{u}) = 0 \tag{8}$$

From Eq. (2c), we find

$$\frac{\partial\Delta T}{\partial t} = \lambda\nabla^2\Delta T \tag{9}$$

Finally linearizing Eq. (2a) about the stationary state yields

$$\begin{aligned}\dot{\vec{u}} &= -\frac{\vec{\nabla}\delta P}{\rho_s} + \frac{\vec{\nabla}P_s}{\rho_s}\frac{\delta\rho}{\rho_s} + \nu\nabla^2\dot{\vec{u}} \\ &= -\frac{\vec{\nabla}\Delta P}{\rho_s} - \frac{g}{\rho_s}(\vec{\nabla}\xi_z)\rho_s - \xi_z\hat{k}g\alpha\beta - \hat{k}g\frac{\Delta\rho}{\rho_s} \\ &\quad + \xi_z\hat{k}g\alpha\beta + \nu\nabla^2\dot{\vec{u}}\end{aligned} \tag{10}$$

Using Eq. (2b), one arrives at $\vec{\nabla}\cdot\vec{\xi} = -\Delta\rho/\rho_s$ and that reduces Eq. (10) to

$$\dot{\vec{u}} = \frac{\vec{\nabla}(\Delta P)}{\rho_s} + \nu\nabla^2\vec{u} = c_s^2\frac{\vec{\nabla}(\Delta\rho)}{\rho_s} + \nu\nabla^2\vec{u} \tag{11}$$

Taking a divergence leads to

$$\begin{aligned}\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{u}) &= -c_s^2\frac{\nabla^2(\Delta\rho)}{\rho_s} + c_s^2\frac{\vec{\nabla}\rho_s}{\rho_s^2}\cdot\vec{\nabla}(\Delta\rho) + \nu\nabla^2(\vec{\nabla}\cdot\vec{u}) \\ &= -\frac{c_s^2}{\rho_s}\nabla^2(\Delta\rho) + c_s^2\frac{\alpha\beta}{\rho_s}\frac{\partial(\Delta\rho)}{\partial z} + \nu\nabla^2(\vec{\nabla}\cdot\vec{u})\end{aligned} \tag{12}$$

Combining with Eq. (8) we get the wave equation

$$\begin{aligned}\frac{\partial^2}{\partial t^2}(\Delta\rho) &= (c_s^2\nabla^2 - c_s^2\alpha\beta\frac{\partial}{\partial z})\Delta\rho - \overline{\rho}\nu\nabla^2(\vec{\nabla}\cdot\vec{u}) \\ &= (c_s^2\nabla^2 - c_s^2\alpha\beta\frac{\partial}{\partial z})\Delta\rho + \nu\nabla^2\frac{\partial(\Delta\rho)}{\partial t}\end{aligned} \tag{13}$$

For the wave propagating with wave-number $\vec{k}$, $\Delta\rho \propto e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ and we have a dispersion relation

$$\omega^2 = c_s^2k^2 + ic_s^2\alpha\beta k_3 - \nu k^2(i\omega) \tag{14}$$

If the angle between $\vec{k}$ and $\hat{z}$ is θ, then

$$\omega^2 = c_s^2k^2 + ic_s^2\alpha\beta k\cos\theta - i\nu\omega k^2 \tag{15}$$

Solving for k,

$$2k = \frac{-ic_s^2\alpha\beta\cos\theta \pm \sqrt{-c_s^4\alpha^2\beta^2\cos^2\theta + 4\omega^2(c_s^2 - i\nu\omega)}}{c_s^2 - i\nu\omega} \tag{16}$$

for low viscosity, i.e., $\frac{\nu\omega}{c_s^2} \ll 1$

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