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Directed selective-tunneling of bosons with periodically modulated interaction

Gengbiao Lu [a,b], Li-Bin Fu [b], Wenhua Hai [c], Mingliang Zou [c], Yu Guo [a]

[a] Department of Physics and Electronic Science, Changsha University of Science and Technology, Changsha 410004, China
[b] Institute of Applied Physics and Computational Mathematics, Beijing 100088, China
[c] Department of Physics, Hunan Normal University, Changsha 410081, China

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ABSTRACT

We study the tunneling dynamics of bosons with periodically modulated interaction held in a triple-well potential. In high-frequency approximation, we derive a set of reduced coupled equations and the corresponding Floquet solutions are obtained. Based on the analytical results and their numerical correspondence, the directed selective-tunneling effect of a single atom is demonstrated when all bosons are prepared in middle well initially. A scheme for separating a single atom from N bosons is presented, in which the atom can be trapped in right or left well by adjusting the modulation strength.

1. Introduction

The coherent manipulation for a quantum system subjected to an external field has been an attractive subject in recent years in both theoretical and experimental physics [1,2]. Coherent control of quantum tunneling for a periodically driven system is one of the most important technologies due to its many applications [3], such as quantum device [4], artificial magnetic fields [5], and quantum information processing [6,7], etc. One of the recent topics in the quantum control of tunneling dynamics is the effect known as coherent destruction of tunneling (CDT) [8], namely, when the strength and frequency of driving force are chosen appropriately, a particle initially located in one of two wells never tunnels to the other. The CDT is based on the fast modulation of level unbalance and the corresponding effect has been verified experimentally [9,10]. Then, a selective CDT effect was found numerically in a driven quantum-dot array [11], in which the quantum tunneling between dots can be suppressed selectively. Such an effect has been demonstrated analytically in a driven tight-binding chain [12]. Further, the selective CDT effect has been introduced to realize a directed-motion scheme of atoms held in a driven one-dimensional bipartite lattice [13,14].

It is well-known that the sign and strength of the s-wave scattering length of interacting cold atoms can be adjusted by using magnetic or optical Feshbach resonances [15]. This technique has been used extensively [16] and many interesting phenomena were demonstrated in the framework of mean-field and Bose–Hubbard models. Such as stable Bloch oscillations [17], self-confinement of two- and three-dimensional Bose–Einstein condensates (BECs) without an external trap [18] and the generation of nonground-state BECs [19]. In a two-mode Bose–Hubbard model, a butterfly pattern of Floquet spectrum is displayed based on the double-kicked modulation of atomic interaction [20]. And in Ref. [21], Gong, Molina and Hänggi have proposed a many-body CDT effect by the periodic modulations of atomic interaction, in which only an arbitrarily, a priori prescribed atoms are allowed to participate in the tunneling process between double wells. An optical realization of the corresponding phenomenon was presented based on light transport in engineered waveguide arrays [22]. Further, the double-well model has been extended to an optical lattice system for the ultracold atoms with periodically modulated interaction [23]. An effective Hubbard-like model was presented, which includes a nonlinear hopping that depends on the difference of occupations at neighboring sites. The rich physics introduced by this hopping were discussed, such as pair superfluid phases, exactly defect-free Mott-insulator states, pure holon, doublon superfluids and quantum Peierls phase, etc.

E-mail address: gengbiaoluu@163.com (G. Lu).

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Recently, the tunneling dynamics of cold atoms held in a triple-well potential have attracted substantial interest and were investigated extensively. Such as stimulated Raman adiabatic transport [24], the transistor-like effect [25] and the effect of dipole–dipole interaction [26,27], etc. In this paper, we further consider the tunneling dynamics of bosons held in a triple-well potential and we are interested in the quantum manipulation of tunneling dynamics based on the periodical modulation of atomic interaction. In our work, we choose bias potential $\varepsilon_0 = 0.5\omega$ and time-independent interaction $U_0 = 0.5\omega/(N-1)$ with ω, N being the modulating frequency and number of bosons, respectively. Under high-frequency approximation, we obtain a set of truncated coupled equations that relate to the subspace spanned by Fock states $\{|0, N-1, 1\rangle, |0, N, 0\rangle, |1, N-1, 0\rangle\}$. When initial state is located in this subspace, we obtain a set of analytical Floquet solutions and the corresponding superposition states. Based on these analytical results, the directed selective-tunneling effect of a single boson is demonstrated, in which the good correspondence is exhibited between analytical and numerical results. It is shown that a single atom can be separated from N bosons and trapped in right or left well by adjusting the modulation strength. The corresponding result presented in our work may be useful in the design of atomic devices [4,6,25,28].

2. Floquet solutions under high-frequency approximation

We consider a system described by the three-mode Bose–Hubbard Hamiltonian, which is realized physically by bosons trapped in a triple-well potential. We consider the interaction strength is modulated periodically in time and the system is described by corresponding Hamiltonian as [21–23]

$$\hat{H}(t) = -\Omega \sum_{\langle k,l\rangle} \left(\hat{c}_k^\dagger \hat{c}_l + \hat{c}_l^\dagger \hat{c}_k\right) + \frac{U(t)}{2} \sum_{k=1}^{3} \hat{c}_k^\dagger \hat{c}_k^\dagger \hat{c}_k \hat{c}_k + \varepsilon_0 \left(\hat{c}_1^\dagger \hat{c}_1 - \hat{c}_3^\dagger \hat{c}_3\right), \tag{1}$$

where $\hat{c}_k^\dagger$ ($\hat{c}_k$) creates (annihilates) an atom in the well k. $\Omega > 0$ is the couplings between nearest-neighbor wells and ε_0 is the potential bias along the triple-well axis. The on-site interaction between atoms is characterized by $U(t) = U_0 + U_1 \cos(\omega t)$, which can be controlled by using suitable Feshbach resonances [15].

In our paper, we have set $\hbar = 1$ and $U_0, U_1, \varepsilon_0, \omega$ and Ω are in units of reference frequency ω_0 on the order of 10^2 s^{-1} [29], and the time t has been normalized in units of ω_0^{-1}. To study tunneling dynamics of bosons held in triple-well system, we introduce the Fock basis $|n_1, n_2, N-n_1-n_2\rangle$ with n_1, n_2 and $N-n_1-n_2$ being the number of atoms in the left, middle and right wells, respectively. In this paper, we consider the total number of atoms N is a constant. On the basis of Fock states, the corresponding quantum state $\Psi(t)$ can be expanded as $|\Psi(t)\rangle = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} a_{n_1,n_2}(t) |n_1, n_2, N-n_1-n_2\rangle$, where $a_{n_1,n_2}(t)$ denote the time-dependent probability amplitudes that obey the normalization condition $\sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} |a_{n_1,n_2}(t)|^2 = 1$. Inserting Eq. (1) and the expanded expression of $|\Psi(t)\rangle$ into Schrödinger equation $i\frac{\partial \Psi(t)}{\partial t} = H(t)\Psi(t)$ results in a set of coupled equations of $a_{n_1,n_2}(t)$ with equation number $\zeta = (N+1)(N+2)/2$.

It is very difficult to obtain the exact solutions of all coupled equations because of the periodically varying coefficients. However, the coherent manipulation of tunneling dynamics can be investigated analytically in high-frequency approximation with $\omega \gg \Omega$. We introduce a set of slowly varying functions $b_{n_1,n_2}(t)$ through the transformation [30] $a_{n_1,n_2}(t) = b_{n_1,n_2}(t) \exp\{-i \int_0^t [0.5U(t)(n_1(n_1-1) + n_2(n_2-1) + (N-n_1-n_2)(N-n_1-n_2-1)) + \varepsilon_0(2n_1+n_2-N)]dt\}$ with $|a_{n_1,n_2}(t)|^2 = |b_{n_1,n_2}(t)|^2$, which leads to that the high-frequency oscillating modulation will be contained in the phase factors. Resembling the fractional photon resonance effect [31], we set the parameters $\varepsilon_0 = 0.5\omega$, $U_0 = 0.5\omega/(N-1)$, and a set of coupled equations of $b_{n_1,n_2}(t)$ can be obtained as

$$\begin{aligned}
i\dot{b}_{0,N-1}(t) = &-\sqrt{N}\Omega b_{0,N}(t) e^{-i[\omega t + U_1(N-1)\sin(\omega t)/\omega]} \\
&- \sqrt{N-1}\Omega b_{1,N-2}(t) e^{-i[\frac{\omega t}{2(N-1)} - U_1(N-2)\sin(\omega t)/\omega]} \\
&- \sqrt{2(N-1)}\Omega b_{0,N-2}(t) e^{i[\omega t - \frac{\omega t}{N-1} + U_1(N-3)\sin(\omega t)/\omega]}, \\
i\dot{b}_{0,N}(t) = &-\sqrt{N}\Omega b_{0,N-1}(t) e^{i[\omega t + U_1(N-1)\sin(\omega t)/\omega]} \\
&- \sqrt{N}\Omega b_{1,N-1}(t) e^{i[U_1(N-1)\sin(\omega t)/\omega]}, \\
i\dot{b}_{1,N-1}(t) = &-\sqrt{N}\Omega b_{0,N}(t) e^{-i[U_1(N-1)\sin(\omega t)/\omega]} \\
&- \sqrt{N-1}\Omega b_{1,N-2}(t) e^{i[\omega t - \frac{\omega t}{2(N-1)} + U_1(N-2)\sin(\omega t)/\omega]} \\
&- \sqrt{2(N-1)}\Omega b_{2,N-2}(t) e^{-i[\frac{\omega t}{N-1} - U_1(N-3)\sin(\omega t)/\omega]},
\end{aligned} \tag{2}$$

where only three coupled equations are presented, in which the probability-amplitude functions $b_{0,N-1}(t)$, $b_{0,N}(t)$, $b_{1,N-1}(t)$, $b_{0,N-2}(t)$, $b_{1,N-2}(t)$ and $b_{2,N-2}(t)$ correspond to states $|0, N-1, 1\rangle$, $|0, N, 0\rangle$, $|1, N-1, 0\rangle$, $|0, N-2, 2\rangle$, $|1, N-2, 1\rangle$ and $|2, N-2, 0\rangle$, respectively. By using Fourier expansion $\exp[\pm i(n\omega t + x\sin(\omega t))] = \Sigma_{n'=-\infty}^{\infty} \mathcal{J}_{n'}(x) \exp[\pm i(n+n')\omega t]$ with $n = 0, 1$ and under the high-frequency approximation, we can neglect these rapidly oscillating terms of the Fourier expansion with $n \pm n' \neq 0$. Simultaneity, these functions oscillating rapidly such as $e^{-i\frac{\omega t}{N-1}}$ and $e^{-i\frac{\omega t}{2(N-1)}}$ in differential equations (2) can be replaced by their average value of zero in the short time interval $2\pi/\omega$ when $\omega \gg 2(N-1)$ [32]. Thus, in high-frequency approximation, the set of coupled equations of $b_{n_1,n_2}(t)$ can be effectively truncated as

$$\begin{aligned}
i\dot{b}_{0,N-1}(t) &= -J_1 b_{0,N}(t), \\
i\dot{b}_{0,N}(t) &= -J_1 b_{0,N-1}(t) - J_2 b_{1,N-1}(t), \\
i\dot{b}_{1,N-1}(t) &= -J_2 b_{0,N}(t).
\end{aligned} \tag{3}$$

In Eq. (3), the effective couplings are given as $J_1 = \sqrt{N}\Omega \mathcal{J}_{-1} \times [(N-1)U_1/\omega]$ and $J_2 = \sqrt{N}\Omega \mathcal{J}_0[(N-1)U_1/\omega]$ with $\mathcal{J}_n(x)$ being the n-order Bessel function of x. Here the effective couplings depend on the modulating parameters, number of atom. And the zeroth- and first-order Bessel functions emerge in the effective couplings resulting from appropriate bias ε_0 and interaction U_0. The different order Bessel function will result in asymmetric tunneling dynamics in the subspace spanned by Fock states $\{|0, N-1, 1\rangle, |0, N, 0\rangle, |1, N-1, 0\rangle\}$ when initial state is prepared in this subspace.

Setting $b_{n_1,n_2} = B_{n_1,n_2} e^{-irt}$ with B_{n_1,n_2} and r being constants and inserting such a form of b_{n_1,n_2} into Eq. (3), the constant r can be obtained as $r_1 = 0$, $r_{2,3} = \pm\sqrt{J_1^2 + J_2^2}$. It is well-known that a quantum state of periodically driven system can be described by $\Psi(t) = \phi(t)e^{-iEt}$ based on the Floquet theorem [33], in which the Floquet state $\phi(t+T) = \phi(t)$ with T and E being the period of Eq. (1) and Floquet quasienergies, respectively. Based on the transformation relation between functions $a_{n_1,n_2}(t)$ and $b_{n_1,n_2}(t)$ and the expression $b_{n_1,n_2} = B_{n_1,n_2} e^{-irt}$, the Floquet energies can be constructed as $E_1 = k$, $E_{2,3} = \pm\sqrt{J_1^2 + J_2^2} + k$ with $0 \le k = (N/4 - m)\omega < \omega$ and $m = 0, 1, 2, \ldots$. The constant B_{n_1,n_2} can be obtained easily from Eq. (3) and the corresponding Floquet states $\phi(t)$ are constructed as

$$\phi_1(t) = \frac{1}{\sqrt{J_1^2 + J_2^2}} \times \left[-J_2 e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t) - i(m-1)\omega t} |0, N-1, 1\rangle\right.$$