Physics Letters A 379 (2015) 962–967

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Non-linear instability of a thin film flowing down a cooled wavy thick wall of finite thermal conductivity

L.A. Dávalos-Orozco

Instituto de Investigaciones en Materiales, Departamento de Polímeros, Universidad Nacional Autónoma de México, Ciudad Universitaria, Circuito Exterior S/N, Delegación Coyoacán, 04510 México DF, Mexico

ARTICLE INFO

Article history:
Received 2 December 2014
Received in revised form 19 January 2015
Accepted 20 January 2015
Available online 22 January 2015
Communicated by C.R. Doering

Keywords:
Thin liquid film
Inclined plane
Deformed thick wall
Marangoni effect
Thermocapillarity
Cooled from the wall

ABSTRACT

The flow of a thin film down a vertical cold thick wall with finite thermal conductivity is investigated under the lubrication approximation. It is shown that, despite the cooling from the wall, it is possible to find a new flow instability. That is, the free surface response to the wall deformation increases its amplitude with the negative Marangoni number. This amplitude growth is independent from the evolution of the time-dependent perturbations imposed on the free surface which, in contrast, are stabilized by cooling from the wall. However it is demonstrated that, even in this case, spatial resonance (see Dávalos-Orozco, 2007, 2008) is more effective to stabilize the time-dependent perturbations. From the results it is evident that these effects are possible only when the magnitudes of the thicknesses ratio and the thermal conductivities ratio are small.

1. Introduction

In real world applications, like surface coating and cooling, the thin liquid films are laid on walls with finite thickness and thermal conductivities. Therefore the recent interest in the investigation of the stability of these films under flow. The motivation for taking different thermal and mechanical wall conditions varies according to the goal of the problem.

Oron et al. [1] investigate evaporative instabilities of thin films. They need to introduce the thickness of the wall to eliminate singularities at the rupture point. Kabova et al. [2] include the thickness of the wall in order to introduce a wall surface topography which is related to experimental settings. Gambaryan-Roisman [3] investigates the stability of a thin film on a wall with non-uniform thermal conductivity. The results are obtained proposing a relation between this non-uniformity and the thickness of the wall. Gambaryan-Roisman and Stephan [4] investigate the effect of longitudinal topography of a thick wall in the formation of rivulets. They include the Lennard-Jones potential in their calculations.

The above mentioned papers motivated a systematic numerical calculation on the nonlinear instability of a thin film flowing down a heated thick wall [5]. In that paper, it is found that the thermal instability is governed by the Marangoni number Ma, the Biot number at the interface of the liquid and the atmosphere (at the free surface) and by d and Q_C. Here, d is the thicknesses ratio of the wall over that of the liquid film and Q_C is the thermal conductivities ratio of the wall over that of the liquid film. These two parameters only appear forming the ratio d/Q_C under the lubrication approximation [6–8] (for a recent review see [9]). The ratio d/Q_C appears in the denominator of the thermocapillary term and consequently its growth has an important stabilizing effect.

The flow down a sinusoidal wall has been investigated under the lubrication approximation by Dávalos-Orozco [10,11]. It is shown that by means of spatial resonance it is possible to stabilize the time-dependent perturbation, even when the fluid is viscoelastic [12]. These particular wall deformations may work as a filter for the perturbations in a finite region of the wall [11] (see a review in [9]). At resonance the wavelength of the wall deformation approaches to that of the time-dependent perturbations and the amplitude of the free surface response increases lowering its valley. Therefore, near to the valley the film is very thin and hence has a local stabilizing effect from which the time-dependent perturbations are not able to recover [10]. The instability of a film flowing down a heated wavy wall was investigated by D'Alessio et al. [13]. Notice that nonlinear results of a Benney type equation under the lubrication approximation are presented in Dávalos-Orozco [9]. It is demonstrated that it is still possible to stabilize the time-dependent perturbations by means of spatial resonance when the wall is an ideal very good conductor.

This last problem has been extended to the case of a heated thick wavy wall with finite thermal conductivity [14]. A nonlinear

E-mail address: ldavalos@unam.mx.

http://dx.doi.org/10.1016/j.physleta.2015.01.018

Nomenclature

A	air jet maximum time-dependent pressure
a	air jet time-dependent pressure dispersion
B	wall deformation amplitude
Bi	Biot number
c	phase velocity
d	d_{wall}/h_0
d_{wall}	wall thickness
$h(x,y,t)$	film local thickness
$H(x,y,t)$	film local perturbation
h_0	unperturbed film thickness
H_h	heat transfer coefficient
k	wavenumber
k_f	fluid heat conductivity
k_{wall}	wall heat conductivity
L	wall wavelength over perturbations wavelength ratio
Ma	Marangoni number
P_p	surface external pressure
Pr	Prandtl number
$Q_c = k_{wall}/k_f$	wall over fluid conductivities ratio
R	Reynolds number
S	scaled surface tension number
T	fluid temperature
$T_{ambient}$	ambient atmosphere temperature
T_L	wall lower face temperature
T_{wall}	wall temperature
T_0	zeroth order fluid temperature
u	velocity x-component
v	velocity y-component
w	velocity z-component
x	non-dimensional x coordinate
y	non-dimensional y coordinate
z	non-dimensional z coordinate
z^*	dimensional z coordinate
Greek	
β	wall inclination angle
Δ	means difference
ε	wave slope smallness parameter
ζ	wall deformation
κ	thermal diffusivity
λ	wavelength
ν	kinematic viscosity
ρ	fluid density
σ	surface tension
Σ	surface tension number
ω	frequency of oscillation

evolution equation of the Benney type is calculated which includes in the denominator of the thermocapillary term an extra spatial variation due to the waviness of the wall. A bump in the free surface response is found near to the valley (the thinnest part) of the wall deformation producing a reduction in the amplitude.

In this paper it will be demonstrated that, when cooling from the wall and the Marangoni number is negative, it is possible to destabilize the thin film free surface response to the wall deformation. The paper is organized as follows. In the next section a brief presentation of the physics of the problem is given along with the evolution equation of the thin film obtained from the basic equations of motion and heat transfer. In Section 3, the numerical results of the evolution equation are exposed graphically and explained in detail. The last section are the conclusions.

2. Thermocapillary flow of a thin film down a cooled wavy thick wall

The system under investigation is a thin film flowing down a cooled wall which has finite thickness and thermal conductivity. The system is sketched in Fig. 1 in non-dimensional form. There, the coordinate system is defined in relation with a flat surface which represents the mean of the wavy deformed wall. Therefore, the x-direction corresponds to the direction of the main velocity of the film. This is perpendicular to the z-direction crossing the film thickness and pointing outwards the fluid film. Assuming a right-handed system, the y-direction is perpendicular to these two. It is assumed that the temperature of the lower face of the wall is T_L and it is located at $z^* = -d_{wall}$ (the star means dimensional), where d_{wall} is the thickness of the wall. The thickness of the film is h_0. It is assumed that the ambient atmosphere above the free surface has a temperature $T_{ambient} > T_L$.

A smallness parameter $\varepsilon = 2\pi h_0/\lambda \ll 1$ is used for the asymptotic expansion of the variables. λ is a representative long wavelength of the perturbations which means that the slope of the free surface deformation is small.

The variables are made non-dimensional by means of h_0 for distance in the z-direction, $\lambda/2\pi$ for distance in the x- and

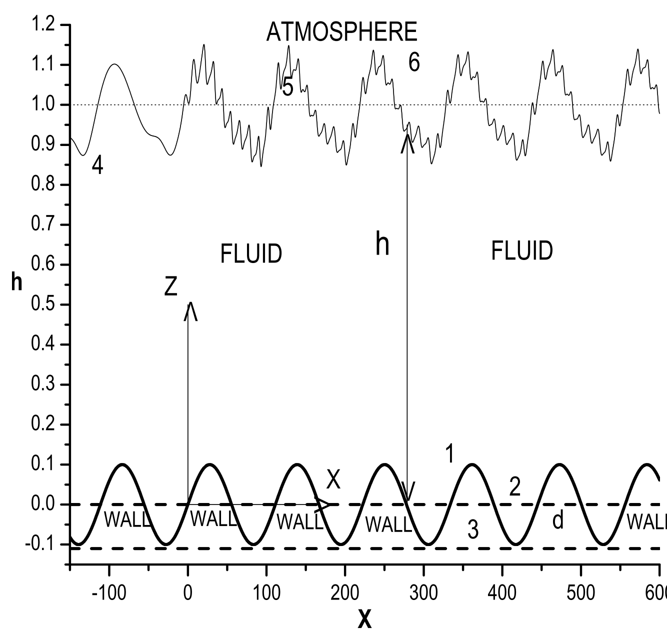

Fig. 1. The thin film and cooled wall assumed vertical. The mean non-dimensional wall thickness is $d = 0.11$. 1) Wall sinusoidal deformation (solid), 2) Mean height of the wall (dashed), 3) Lower side of the wall located at $z = -0.11$ (dashed) with non-dimensional temperature 1, lower than that of the atmosphere above the free surface. 4) Free surface response to the wall deformation, 5) Mean height of the unperturbed free surface (dotted), 6) Time-dependent perturbations excited at $x = 0$ and running on the free surface response. They have a local height $h(x,t)$ with respect to the wall deformation. The largest and smallest thickness of the wall are 0.21 and 0.01, respectively.

y-directions, $h_0\lambda/(\nu 2\pi)$ for time, ν/h_0 for velocity, $\rho\nu^2/h_0^2$ for pressure and $\Delta T = (T_L - T_{ambient}) < 0$ for temperature. The kinematic viscosity and the density are ν and ρ, respectively.

In non-dimensional form the free surface is assumed to be located at $z = \zeta(x,y) + 1$ before the application of a perturbation, where $\zeta(x,y)$ is the wall deformation. When the surface is perturbed the location is set as $z = \zeta(x,y) + h(x,y,t)$ where