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Accelerating nondiffracting beams

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A B S T R A C T

We present a set of beams which combine the properties of accelerating beams and (conventional) diffraction-free beams. These beams can travel along a desired trajectory while keeping an approximately invariant transverse profile, which may be (higher-order) Bessel-, Mathieu- or parabolic-nondiffracting-like beams, depending on the initial complex amplitude distribution. A possible application of these beams presented here may be found in optical trapping field. For example, a higher-order Bessel-like beam, which has a hollow (transverse) pattern, is suitable for guiding low-refractive-index or metal particles along a curve.

1. Introduction

In 1979, Berry and Balazs [1] predicted a new family of solutions to the Schrodinger equation for a free particle: Airy wave packets. Airy wave packets have unique propagation behaviors of non-diffraction and free acceleration. In 2007, Siviloglou et al. predicted theoretically [2] and demonstrated experimentally [3] finite energy Airy wave packets in the optics domain. These optical Airy wave packets, called Airy beams, exhibit the properties of quasi-diffraction-free and free acceleration along a parabolic curve over a certain distance. Since then, related applications including optical manipulation and generation of curved plasma channel [4–6] were also demonstrated. In parallel, various methods are also proposed to generate Airy beams, such as using a spatial light modulator (SLM) [2,4], a continuous transparent phase mask [6], asymmetric nonlinear photonic crystals [7] and surface plasmon polariton fabrication [8]. Usually, for an accelerating beam along a parabolic curve, the transverse profile of the beam is described by the Airy function modulated by a decay factor. However, Bandres [9] found that, under the condition of parabolic acceleration, the equation governing the transverse profile of beam allows for the separation of variables in the parabolic coordinate system. Thus, accelerating parabolic beams are also possible. The fields of such beams exhibit well defined parabolic nodal lines.

Recently, beams with non-parabolic trajectories were also demonstrated in both paraxial regime [10,11] and non-paraxial regime [12–17]. With the method of stationary phase, Greenfield et al. [10] created one-dimensional accelerating beams along arbitrary convex trajectories. The transverse profile of such beams is described by an Airy-like function. In the paraxial condition, the trajectory of an accelerating beam is limited to a small angle, that is, it cannot bend to large angles at which the beam is no longer shape preserving. To overcome this restriction, non-paraxial accelerating beams (NABs) have been identified theoretically and demonstrated experimentally. In two-dimensional case, NABs can be exact solutions of the Helmholtz equation (HE) in different cylindrical coordinate systems [14,15]. Unlike paraxial accelerating beams, which finally break down at large distance, NABs can travel along different planar curves beyond the paraxial limit including circle [14], ellipse and parabola [15]. Moreover, they can bend themselves to a large angle close to 90° — perpendicular to the original direction of propagation. For circular accelerating beams, the dynamic properties of acceleration and diffraction-free occur simultaneously. More recently, circular accelerating beams were generalized to three-dimensional case in the spherical and (prolate) spheroidal coordinate systems [16], and the parabolic coordinate system [17]. Classified by the indices of some special functions, these three-dimensional circular accelerating beams exhibit different transverse modes (in the azimuthal plane). For example, in the spherical coordinated system, the transverse mode is specified by the indices of the associated Legendre function.

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In general, the transverse profile of accelerating beams shows an asymmetry. Recently, Chremmos et al. [18] found a set of accelerating beams which exhibit a zeroth-order Bessel-like property in their transverse profile. This theoretical work was later proved by Zhao et al. [19].

In this paper, we extend the work of Chremmos et al. to beams that have a transverse profile resembling conventional diffraction-free (higher-order) Bessel beams, Mathieu beams or parabolic non-diffracting beams, depending on the initial complex amplitude distribution. We know that these three types of beams are diffraction-free solutions in the cylindrical [20], elliptic cylindrical [21,22] and parabolic cylindrical [23] coordinate systems respectively, which preserve their shapes during propagation.

2. Theoretic consideration

The previous work of Chremmos et al. gave a set of zeroth-order Bessel-like accelerating beams with arbitrary trajectories. The key point of the method used there is to find an initial field distribution in the input plane $Z = 0$

$$u(x, y) = A(x, y)\exp\left[iQ(x, y)\right], \tag{1}$$

whose paraxial Fresnel integral

$$u(X, Y, Z) = \frac{1}{i2\pi Z}\iint u(x, y)e^{i\frac{(X-x)^2+(Y-y)^2}{2Z}}dxdy \tag{2}$$

will give a field distribution in the region $Z > 0$ having a desired accelerating trajectory given by $X = f(Z)$ and $Y = g(Z)$. As in [18], the transverse and longitudinal coordinates are divided by X_0 and $k(X_0)^2$ respectively, where k is the wave number and X_0 an arbitrary constant.

In [18], only the initial phase $Q(x, y)$ is concerned. To avoid confusion, we denote by $Q_0(x, y)$ the initial phase used there. By the stationary phase conditions $\partial_x Q_0 = 0$ and $\partial_y Q_0 = 0$ on the curve, and the integrable condition $\partial_x\partial_y Q_0 = \partial_y\partial_x Q_0$, Chremmos et al. found that the stationary phase points contributing to the point $(f(Z), g(Z), Z)$ on the accelerating curve consist of a circle $C(Z)$ described by

$$(x - x_c)^2 + (y - y_c)^2 = R^2, \tag{3}$$

where $x_c = f - Zf'$, $y_c = g - Zg'$ with the prime denoting the derivative relative to Z, and R is a function of Z. Solving this equation for Z in favor of x and y, and integrating the stationary phase condition $\partial_x Q_0 = 0$ or $\partial_y Q_0 = 0$, the initial phase $Q_0(x, y)$ is found to be

$$Q_0(x, y) = \frac{1}{2}\int_0^Z \left[\left(f'\right)^2 + \left(g'\right)^2 - (R/s)^2\right]ds - \left[(f - x)^2 + (g - y)^2\right]/(2Z). \tag{4}$$

To see that the initial phase distribution $Q_0(x, y)$ gives a zeroth-order Bessel-like accelerating beam, note that for a given Z, the field in the transverse neighborhood of the point $(f(Z), g(Z), Z)$ is mainly contributed by the circle $C(Z)$ in the initial plane. Chremmos et al. [18] showed that the rays emanated from the points on the circle $C(Z)$ interferes constructively to give a field pattern as

$$u(\delta X, \delta Y, Z) = \exp\left[iP(Z)\right] \times \oint_{C(Z)} \exp\left[i(f - x)\delta X + i(g - y)\delta Y\right]dl, \tag{5}$$

where $P(Z)$ is a function of Z [18]. It is recognized that, the right-hand side of (5) is the integral representation of a zeroth-order Bessel function. To yield a higher-order Bessel function, we simply multiply the integrand in the integral in Eq. (5) by a modulation factor $h(\phi) = \exp(im\phi)$, which gives a Bessel function of order m. Furthermore, if we put $h(\phi)$ to be the eigen functions appearing in the integral representation of Mathieu beams or parabolic non-diffracting beams, then the field obtained from (4) will exhibit an elliptic or parabolic geometry. Noting that the integral in (5) is performed over a certain circle $C(Z)$, the introduction of the factor $h(\phi)$ is also confined to this circle, that is, its argument ϕ is a function of Z. Since the function $h(\phi)$ is complex valued, there results an additional phase $\mathrm{Arg}(h(\phi))$ besides $Q_0(x, y)$ given by Eq. (4). Then, the final initial phase and magnitude occurring in Eq. (1) are $Q(x, y) = Q_0(x, y) + \mathrm{Arg}(h(\phi(Z)))$ and $A(x, y) = |h(\phi(Z))|$, where the function $Z(x, y)$ is obtained by solving Eq. (3). We emphasize that by the introduction of the modulation factor $h(\phi)$, the stationary phase condition $\partial_x Q = 0$ and $\partial_y Q = 0$ may not hold on the accelerating curve, implying that the maxima of field may not occur on the curve. However, this is what we expect, since, for example, a higher-order Bessel-like (accelerating) beam must have a transverse profile of donut shape. As pointed out by Chremmos et al. [18], the accelerating beams obtained by this method exhibit a well-defined acceleration behavior only for $Z \le Z_m$, where Z_m is a critical value determined by Eq. (3). For $Q(x, y)$ to be well defined, as in [18], the trajectory beyond Z_m is set to be a straight line tangent to the well defined acceleration curve at the ultimate point $(f(Z_m), Z_m)$.

3. Simulations and discussions

In what follows, we confine the acceleration trajectory to a curve in the $Y = 0$ plane ($g = 0$) and put $R(Z) = Z$ in Eq. (3). Using the above method, we obtain the initial field distribution $u(x, y)$ and substitute it into the Fresnel integral (2) to carry out numerical simulations. We first give an example of the higher-order ($m = 2$) version of the zeroth-order Bessel-like beam along the parabola $f = Z^2/40$ given in Fig. 2 of [18]. As in [18], the initial amplitude is modulated by the Gaussian factor $\exp[-(x^2 + y^2)/900]$. Noting that here (x, y) and (X, Y, Z) are dimensionless coordinates corresponding to the actual coordinate variables (xX_0, yX_0) and $(XX_0, YX_0, kZ(X_0)^2)$ with k being the wave number and X_0 an arbitrary constant. If $X_0 = 100$ μm and the wavelength $\lambda = 1$ μm, an 80×80 transverse region such as Fig. 1(a) would correspond to an actual region of 8 mm × 8 mm, while the longitudinal interval [0, 30] such as Fig. 1(c) would correspond to [0, 188.4 cm]. The phase $Q(x, y)$ and the magnitude $A(x, y)$ are given in Figs. 1(a) and 1(b) where the magnitude $A(x, y)$ is just the Gaussian factor. Compared to the phase distribution of the zeroth-order Bessel-like beam given in [18], the phase here shows some similarities in the outer region. In the central region, however, Fig. 1(a) presents a twisted structure, a typical property of phase vortex. This is reasonable, since on each circle $C(Z)$, a local vortex phase factor $\exp(im\phi)$ exists. Fig. 1(c) describes the dynamics of propagation of the beam in the $Y = 0$ plane, where, as desired, a parabola trajectory (dashed line) is observed. We note that the whole field distribution in the $Y = 0$ plane consists mainly of two curved strips located on two sides of the prescribed parabola trajectory. Figs. 1(d)–1(f) show the transverse profiles at $Z = 5, 15$ and 25, respectively, exhibiting the multiple circular ringed structure with a vanishing field at the center, the typical pattern of a higher-order Bessel beam. This transverse profile, in combination with the parabola trajectory, enables us to obtain a curved donut beam in three-dimensional physical space. From Figs. 1(d)–1(f), we also observe that the transverse profile is approximately invariant. However, we note that the main (central) ring is not uniform. At $Z = 5$, the main ring assumes its maximum roughly at the ten clock position, while at $Z = 15$ and 25, they are approximately at the one