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Pseudo-two-dimensional random dimer lattices

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ABSTRACT

We study the long-time wave transport in correlated and uncorrelated disordered 2D arrays. When a separation of dimensions is applied to the model, we find that the previously predicted 1D random dimer phenomenology also appears in so-called pseudo-2D arrays. Therefore, a threshold behavior is observed in terms of the effective size for eigenmodes, as well as in long-time dynamics. A minimum system size is required to observe this threshold, which is very important when considering a possible experimental realization. For the long-time evolution, we find that for correlated lattices a super-diffusive long-range transport is observed. For completely uncorrelated disorder 2D transport becomes sub-diffusive within the localization length and for random binary pseudo-2D arrays localization is observed.

1. Introduction

The concept of wave localization due to disorder, known as Anderson localization (AL), has been around for quite some time [1], and several reviews with compiled progresses on this topic have been written recently (cf. Refs. [2–4]). This phenomenon appears as a consequence of the destructive interference of multiple scattered waves and has been observed in such different physical contexts as electronics, photonics, Bose–Einstein condensates (see Refs. [5–11]), to name a few. Whenever the corresponding physical system can be modeled with a tight-binding Hamiltonian, with time-invariant potential for non-interacting particles, AL can be found. This is particularly, for example, the case for the propagation of light in evanescently coupled optical waveguide arrays [4]. Here, over recent years, impressive progress in experimental observations in one (1D) and two dimensions (2D) has been made [12,13]. Diverse quantum, and condensed matter, phenomena have been reproduced in these – clean and macroscopic – setups, by using electromagnetic waves [14–16]. Moreover, it has been possible to incorporate controlled disorder during the fabrication of these photonic structures, and different studies concerning AL have been carried out [17,4].

In general, localization properties depend strongly on the dimensionality of the system [18]. In contrast to the three-dimensional (3D) case, in 1D and 2D already the slightest amount of uncorrelated disorder leads to a complete exponential localization for all eigenmodes without any mobility edge, even though the localization length is much larger for 2D systems [19]. But, when correlations are included in the system, the picture changes dramatically [20] and long-range transport may be still possible, even in low dimensions (for a recent review, see [21]). The paradigmatic example in 1D is the *random dimer model* (RDM) [22,23], where the pairing of adjacent on-site energies (dimers), at random positions in the lattice, leads to two-site correlations for an otherwise random binary model. For finite lattices of length N, the RDM shows that, below a certain threshold of the disorder strength, there are $\sim\sqrt{N}$ extended (thus transparent) states, resulting in super-diffusive wave-packet evolution below the threshold, and diffusive transport exactly at the threshold region [22]. These delocalized eigenstates were shown in experiments [24,25], whereas a direct observation of the transport properties was reported only very recently [17].

A two-dimensional rectangular optical waveguide array can be thought as a classical analog to study quantum transport of two interacting particles in an one-dimensional chain, in the context of a Bose–Hubbard model. The 1D problem is mapped to a 2D lattice, where the interaction between particles is described by taking the propagation constants at the lattice diagonal ($n = m$) different to the rest of the lattice [26–28]. Recently [29], it was shown that the interaction between particles could promote a metallic two-particle state in a 1D quasiperiodic lattice, whereas the single-particle (classical) regime presented no transport [30,31]. Therefore, the study of a 2D random dimer lattice presents an

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interesting possibility to observe similar features in a model presenting a well defined transport transition.

In the present work, we will use a special construction of disorder in two dimensions, named *pseudo-2D*, to map our system described by a tight-binding Hamiltonian for non-interacting particles onto 1D chains. This enables us to access the super-diffusive transport properties in such arrays. In Section 2 we will first present the model of pseudo-two-dimensional lattices using a separation ansatz; in Section 3 we use the extension of eigenmodes in smaller lattices to show the threshold behavior and its dependence on system size. In Section 4 we show the super-diffusive transport in the numerical long-time evolution of correlated arrays. We also compare the localization volume of random binary pseudo-2D arrays with 1D lattices. Finally we conclude the present work in Section 5.

2. 2D random dimer model

We start from a 2D discrete linear Schrödinger equation corresponding to a tight-binding Hamiltonian for non-interacting particles, which also describes, e.g., the evolution of the field envelope amplitude of light propagating along the longitudinal direction z in a 2D linear waveguide array [32]:

$$-i\,\mathrm{d}_z\, a_{n,m} = \epsilon_{n,m} a_{n,m} + \Delta_{nm} a_{n,m}, \tag{1}$$

with $\mathrm{d}_z \equiv \frac{\mathrm{d}}{\mathrm{d}z}$ and $\epsilon_{n,m}$ corresponding to the onsite-propagation constant at site $\{n,m\}$. Linear coupling, between nearest-neighbor sites of a square lattice, is defined as $\Delta_{nm} a_{n,m} \equiv C(a_{n,m+1} + a_{n,m-1} + a_{n+1,m} + a_{n+1,m})$, where C represents the hopping constant. Without loss of generality, we set $C = 1$, having in mind its rescaling effect on site energies and the effective propagation time/distance. Now, by restricting our study to the case: $\epsilon_{n,m} \equiv \varepsilon_n + \varepsilon_m$, we make a dimension reduction by means of a separable ansatz: $a_{n,m}(z) = u_n(z) v_m(z)$. Thus, we obtain two *independent* set of equations [32] for each separable dimension:

$$\begin{aligned} -i\,\mathrm{d}_z\, u_n &= \varepsilon_n u_n + (u_{n-1} + u_{n+1}), \\ -i\,\mathrm{d}_z\, v_m &= \varepsilon_m v_m + (v_{m-1} + v_{m+1}). \end{aligned} \tag{2}$$

The propagation constants ε_l $(l = n, m)$ are chosen in random pairs (dimers) as follows:

$$\varepsilon_l = \varepsilon_{l+1} = \begin{cases} \Delta, & \text{if } \kappa \le 1/2, \\ 0, & \text{if } \kappa > 1/2, \end{cases} \tag{3}$$

where κ is chosen randomly in the interval $[0, 1]$. Δ corresponds to the index (energy) contrast, which defines the differences in propagation constants (energies) between two different lattice sites. In the following, we will consider four different cases of disorder realizations:

i. Equally correlated random dimers (ECORADI): $\varepsilon_n = \varepsilon_m$ [sketched in Fig. 1(a)].
ii. Different correlated random dimers (DICORADI): $\varepsilon_n \neq \varepsilon_m$ [sketched in Fig. 1(b)].
iii. A binary case of uncorrelated random monomers (UNCORAM); i.e., the case where the onsite propagation constants in model (1) are chosen randomly between two precise values: 0 or 2Δ.
iv. A binary case of pseudo-2D uncorrelated random monomers (RAMPS), where the onsite propagation constants *for each site* in model (2) are chosen randomly between the precise values: 0 or Δ.

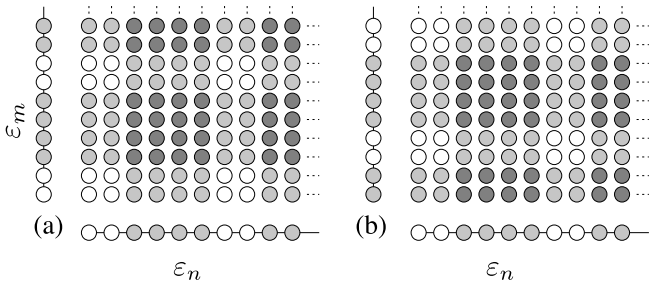

Fig. 1. Sketch of pseudo-2D random dimers: (a) ECORADI and (b) DICORADI. The individual realizations of ϵ_n and ϵ_m are shown on both axis. ○ $\epsilon_{n,m} = 0$, ● $\epsilon_{n,m} = \Delta$, ● $\epsilon_{n,m} = 2\Delta$.

3. Eigenmodes size

To understand the fundamental properties of different lattices, it is crucial to get some information about the eigenmodes of every system. By analyzing their spatial localization features, we could gain a good insight of possible transport properties on the particular lattice. In order to investigate the spatial extension of a wave-packet in a square lattice of $N \times N$ sites with fixed boundary conditions, we define the normalized participation ratio as

$$R \equiv \frac{P^2}{N^2 \sum_{n,m} |u_{n,m}|^4}, \tag{4}$$

with $P = \sum_{n,m} |u_{n,m}|^2$ being a conserved quantity of model (1). In an optical waveguide array context, P corresponds to the optical power (in other contexts as BECs, this quantity is usually named as Norm or Number of particles). The participation ratio R is a very useful quantity [33,34], which helps us to identify the number of effectively excited sites of a given profile, it can be understood as well as an effective occupied area of the profile. For a highly localized wave packet, R approaches $1/N^2$, and tends to 1 for the case of a completely homogeneous array excitation. We use it here to calculate the extension of the N^2 eigenmodes of a 2D square array with given disorder distribution.

Following the arguments presented in Ref. [22], in a 1D array of N lattice sites governed by the RDM, a fraction of $\sqrt{N}$ eigenmodes are extended over the whole lattice, as long as $\Delta \le 2$. Thus, long-range transport is possible below a certain threshold. Therefore, in a separable pseudo-2D array described by Eqs. (2), with system size $N \times N$, we expect the same behavior, but for $\sqrt{N^2} = N$ states. In Fig. 2, we plot $\langle R_N \rangle$ versus the contrast degree Δ. R_N is defined as the average participation ratio for the N most extended (largest R-value) eigenmodes, of a given realization, for a given Δ. Then, $\langle R_N \rangle$ is obtained by averaging over 100 realizations for each Δ-value, for different system sizes (N^2). For this computation, we choose systems sizes with $N : \{20, 60\}$, in order to trace the emergence of the threshold behavior. This is done in order to estimate the smallest N^2-value required to observe the predicted critical phenomenology, which is important when thinking on an experimental observation of the present results.

The participation ratio for all cases (i)–(iv) is displayed in Figs. 2(a)–(d). For ECORADI [see Fig. 2(a)], the values of $\langle R_N \rangle$ first decrease to some kind of plateau at $\Delta = 2$, with a value of $\langle R_N \rangle > 0.3$. This corresponds roughly to an average size of the eigenmodes larger than 30% of the lattice size, what actually implies very delocalized states [for the ordered case ($\Delta = 0$) and for very small Δ extended linear modes cover around 45–50% of the lattice]. For $\Delta > 2$, $\langle R_N \rangle$ drops abruptly, especially for larger lattices. This phenomenon can be clearly attributed to the threshold behavior of the RDM [17,22,23], what is highly dependent on the system size. Therefore, in order to indeed observe a larger number of modes