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Enlargement of locally resonant sonic band gap by using composite plate-type acoustic metamaterial

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ABSTRACT

We numerically investigate the propagation characteristics of Lamb waves in composite plate-type acoustic metamaterial constituted of one-side cylindrical stubs deposited on a two-dimensional binary locally resonant phononic plate. Numerical results show that, with the introduction of composite plate-type acoustic metamaterial, locally resonant band gap shifts to lower frequency, and a significant enlargement of the relative bandwidth by a factor of 3 can be obtained, compared to one-side locally resonant stubbed plates. We show that the band gap enlargement is attributed to the coupling between the local resonant Lamb modes of two-dimensional phononic plate and the resonant modes of the stubs.

1. Introduction

Phononic crystals (PCs) and locally resonant acoustic/elastic metamaterials have attracted considerable attention over the last two decades due to their abundant physical characteristics, such as the existence of phononic band gaps (BGs) [1–8]. In a phononic crystal, BGs are generated by Bragg scattering for which wavelengths are on the scale of the structure's periodicity. The width and position of the Bragg band gaps depend strongly on the elastic parameters' contrast between the scattering and the host material, as well as the geometric parameters and shape of the inclusions [9–15]. Significantly, Bragg band gaps in the low frequency range are difficult to be obtained as the Bragg scattering mechanism requires that the dimension of the PCs should be in the same order of magnitude with the wavelength of the elastic waves, which limited the application of PCs in low-frequency range.

To overcome the limitation of the Bragg band gaps, Liu et al. [16] reported the acoustic waves propagating in locally resonant acoustic metamaterials composed of periodic coated lead balls immersed in an epoxy matrix, and obtained band gaps two orders of magnitude smaller than that of the Bragg gaps. Based on their pioneer work, large efforts have been concentrated on the formation mechanisms of locally resonant band gaps and the influence factors investigation [17–32]. Indeed, BGs in locally resonant acoustic/elastic metamaterials are generated by the coupling between the modes propagating in the matrix and the localized resonant modes of the scatterers. The latter have a zero group velocity (flat band) and the elastic energy is completely confined within the resonators (high quality factor), resulting in a very narrow band gap and a sharp resonant frequency spectrum. Consequently, how to enlarge the locally resonant band gaps is of great importance to promote the application of locally resonant acoustic/elastic metamaterials in vibration and noise control in low-frequency range. Ma et al. [33] revisited two previously suggested structures, which are constructed by periodically drilling holes on elastic plate and then filling them with the rubber-coated masses, or just by periodically stubbing the rubber rods with mass cap on the plate. It can be found that the width of the BG is limited by the worse overlapping of the partial gaps. Based on the understanding, a new structure with the three-layered spherical resonant units is proposed. Numerical results show that, making use of such kind of resonant units, a large sub-wavelength full band gap can be opened. Xiao et al. [34] studied the flexural vibration band gaps in a thin plate with two-dimensional ternary locally resonant structures, i.e. a thin epoxy plate containing a periodic square array of lead discs hemmed around by rubber. However, these two structures are constituted of rubber-coated masses filled in a homogeneous thin plate and the formation mechanisms of band gap in these two structures are mainly attributed to the local resonant of the resonance units. Zhu [35] showed that effective zero refractive index in locally resonant acoustic metamaterial can be constructed by the resonant unit-cells with coherent degenerate monopole-dipole momenta. Assouar et al. [36] reported the theoretical analysis of the enlargement of locally resonant acoustic band gap in

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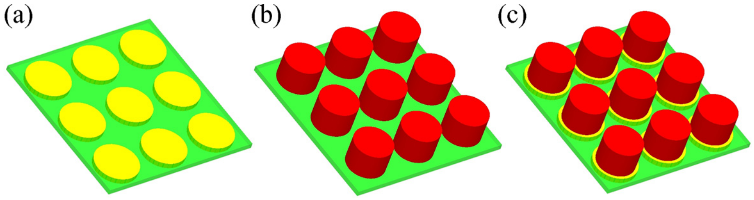

Fig. 1. Illustration of the concept of composite plate-type acoustic metamaterial. (a) Two-dimensional binary locally resonant phononic plate (consisting of an array of rubber inclusions embedded in epoxy matrix with a finite thickness). (b) One-side locally resonant stubbed plate (consisting of a periodic array of rubber stubs on a homogeneous epoxy plate). (c) Composite plate-type acoustic metamaterial (consisting of a periodic array of rubber stubs on a two-dimensional binary locally resonant phononic plate).

two-dimensional sonic crystals based on a double-side stubbed plate. A significant enlargement of the relative bandwidth by a factor of 2 compared to the classical one-side stubbed plates was obtained and discussed. Bilal et al. [37] investigated the dispersion characteristics of locally resonant elastic metamaterials formed by the erection of pillars on the solid regions in a plate patterned by a periodic array of holes. They showed that the trampoline effect caused subwavelength bandgaps to increase in size by up to a factor of 4. Significantly, current research on locally-resonant acoustic metamaterials is mainly focused on investigation of band gaps in phononic crystals consisting of cylindrical stubs deposited on a homogeneous thin plate. However, relatively little attention has been paid to investigating the propagation characteristics of Lamb waves in composite plate-type acoustic metamaterial constituted of one-side cylindrical stubs deposited on a two-dimensional binary locally resonant phononic plate.

In this Letter, we propose an original composite plate-type locally resonant acoustic metamaterial constituted of one-side cylindrical stubs deposited on a two-dimensional binary locally resonant phononic plate and present the theoretical investigation of the propagation characteristics of Lamb waves in the structure. The dispersion relations and the displacement fields of the eigenmodes are calculated by using the finite element method. We show that a significant enlargement of the relative bandwidth by a factor of 3 compared to two-dimensional binary locally resonant phononic plates and the classical one-side locally resonant stubbed plates can be obtained. By means of this composite structure, locally resonant band gaps with large bandwidth in the low-frequency range could be achieved simultaneously.

2. Numerical results and discussion

As shown schematically in Fig. 1, the physical model considered in this study (Fig. 1(c)) is constructed by depositing the rubber cylinder stubs (Fig. 1(b)) squarely onto the surface of a two-dimensional binary locally resonant phononic plate, which consist of an array of rubber inclusions embedded in epoxy matrix with a finite thickness (Fig. 1(a)). In the unit cell, the lattice constant, the plate thickness and the radius of the circular fillers are denoted by a, e and r_0 respectively. The radius and height of the rubber cylinder stubs are denoted by r and h respectively. The material parameters are chosen as follows: the density of soft rubber is assumed as 1300 kg/m^3, and that of epoxy is 1180 kg/m^3. The phase velocities of longitudinal and transverse waves in soft rubber are $c_L = 33$ m/s and $c_T = 5$ m/s, respectively. In epoxy, $c_L = 2534$ m/s and $c_T = 1157$ m/s, respectively.

To investigate the propagation characteristics of Lamb waves in the proposed composite plate-type acoustic metamaterials, a series of dispersion relations are calculated using a finite element method which has been proven to be an efficient method for obtaining the phononic crystal dispersion curves in previous works [25–27]. For the mesh elements, we chose the default second-order Lagrange element provided by Comsol Multiphysics 3.5*a*. The default second-order Lagrange elements add additional degrees of freedom on midpoint and interior nodes in the mesh elements. These added degrees of freedom provide a more accurate solution but also require more memory due to the reduced sparsity of the discredited system. Stress-free boundary conditions are used for free surfaces and the periodic boundary conditions are used for the interfaces between the nearest unit cells according to the Bloch theorem. Because of the periodicity of the structure, only one unit cell is considered in the calculation. By varying the wave vector in the first irreducible Brillouin zone, the Bloch calculation gives the eigenfrequencies and the corresponding eigenvectors, and then the dispersion relations, as well as the eigenmodes, can be obtained.

To illustrate the propagation characteristics of Lamb waves in the proposed composite plate-type locally resonant acoustic metamaterial, some numerical calculations are carried out. The geometrical parameters are chosen as follows: the lattice constant is $a = 10$ mm, the plate thickness is $e = 1$ mm, the radius of the circular fillers is $r_0 = 4$ mm, the radius and height of the rubber cylinder stubs are denoted by $r = 3.5$ mm and $h = 5$ mm respectively. Figs. 2(a)–(c) illustrate the dispersion relations of two-dimensional binary locally resonant phononic plates and one-side locally resonant stubbed plates as well as composite plate-type locally resonant acoustic metamaterial respectively. The vertical axis is the normalized frequency $\omega a/c_T$, where ω is the wave frequency and c_T is the transversal wave speed of epoxy. The horizontal axis is the reduced wave vector taken along the first irreducible Brillouin zone. One can observe that, not only the traditional plate modes such as the shear, symmetric and antisymmetric Lamb modes, but also lots of flat modes can be found in the band structures. The band gaps open between the 4th and 5th resonant modes of the disks and stubs. The band gaps opened in these systems are located about two orders lower than the Bragg scattering mechanism in the frequency region. In addition, with the introduction of composite plate-type locally resonant acoustic metamaterial, local resonance enhances, locally resonant band gap shifts to lower frequency, the relative bandwidth of the complete band gap is significantly enlarged. Indeed, this relative bandwidth is equal to 8% and 10% for two-dimensional binary locally resonant phononic plate and one-side locally resonant stubbed plate respectively, while it is equal to 30% for composite plate-type locally resonant acoustic metamaterial. The relative bandwidth $\Delta\omega/\omega_c$ is the ratio between the band gap width and the center frequency of the band. It can be clearly observed that the introduction of composite plate-type acoustic metamaterial gives rise to a significant enlargement of the relative bandwidth by a factor of 3 compared to two-dimensional binary locally resonant phononic plates and one-side locally resonant stubbed plates.

To intuitively illustrate the mechanism of locally resonant band gap enlargement in composite plate-type acoustic metamaterials, the displacement fields of the eigenmodes are carried out. Fig. 3(a) shows the displacement fields of the eigenmodes of two-dimensional binary locally resonant phononic plate. It can be found that the displacement field is almost concentrated in the circular fillers and manifests the local resonance modes of the rubber inclusions. Fig. 3(b) illustrates the displacement fields of the eigenmodes of one-side locally resonant stubbed plate. One can observe that, the opening of the local resonance band gap can be