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Linear response of a pre- and post-selected system to an external field

Masashi Ban

Graduate School of Humanities and Sciences, Ochanomizu University, 2-1-1 Ohtsuka, Bunkyo-ku, Tokyo 112-8610, Japan

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ABSTRACT

Linear response to an external field is studied for a quantum system with pre- and post-selection. Effects of an external field on strong and weak values of a system observable are found. The external field applied after the measurement of the observable influences the linear response of the system through post-selection. A time-symmetric property in the linear response is found.

1. Introduction

A measurement process is indispensable for gaining information on a quantum system. As a result, a measured system changes its state irreversibly into another one which depends on the measurement outcome [1,2]. This characteristic feature of a quantum system is called the state reduction. A closed system to be measured is prepared in an initial state and it evolves reversibly with time. Quantum measurement is performed on the system at some time and the state reduction takes place. When the system is irrelevant after the measurement, the overall time evolution of the system is clearly asymmetric or irreversible due to the state reduction caused by the measurement. However, Aharonov, Bergmann and Lebowitz [3] have shown that time symmetric formulation of quantum mechanics is possible. In their formulation, the measured system evolves again with time until post-selection of the system is performed at final time. The symmetry can be recovered if measurement of an observable is done between pre-selection (initialization) and post-selection. In a usual theory, only pre-selection is considered. When both pre-selection and post-selection are performed, quantum systems exhibit many interesting properties [4–10].

The linear response theory developed by Kubo provides a powerful tool for investigating properties of a physical system [11,12]. It has been applied to various kinds of phenomena from solid state physics [12] to quantum optics [13,14]. In the linear response theory, we examine how an average value of a system observable changes under the influence of a weak external field. The response of the average value depends only on the external field applied during the time evolution of the system from initial time to measurement time. It does not depend on the external field applied after the measurement is completed. However, when post-selection is performed on the system, it is possible that the linear response is influenced by the external field applied during the time evolution of the system from measurement time to final time. Then some time-symmetric property can be expected in the linear response.

The linear response theory is one of the most successful theories in non-equilibrium statistical mechanics [11], and the symmetric formulation of quantum mechanics is of great importance in the basis of quantum mechanics [3]. Hence it is interesting to consider the linear response theory within the framework of the time-symmetric quantum mechanics. It is expected that some time-symmetric property can be found even in the presence of disturbance caused by an external field. The linear response theory has been applied to the weak measurement [15]. The linear response of a pointer observable of a measuring device to a relevant system has been considered, where a weak value of a system observable plays a role of the external field to the measuring device and time-symmetric property has not been discussed. The situation considered in this paper is quite different from that in [15]. We will consider a compound system which consists of three parts: one is a relevant system, another is a measuring device which is strongly or weakly coupled to the relevant one, and the other is an external field which is applied for investigating the linear response of an observable of the relevant system.

In this work, we apply the linear response theory to a quantum system, where both pre-selection and post-selection are performed. We consider strong and weak measurement to obtain an

E-mail address: ban.masashi@ocha.ac.jp.

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average value of a system observable. In Section 2, we briefly summarize the time evolution of strong and weak values of an observable of a pre- and post-selected system. In Section 3, we drive the linear responses of strong and weak values to a weak external field and we discuss their properties. In particular, we investigate time-symmetric property in the linear response. In Section 4, we provide concluding remarks.

2. Strong and weak values in a pre- and post-selected system

We suppose that a physical system is initially prepared (or pre-selected) in a quantum state described by a density matrix $\hat{\rho}_i$ at initial time t_i. The system may consist of two parts; one is a relevant system and the other is a surrounding environment. The time-evolution of the whole system is determined by a Hamiltonian $\hat{H}(t)$ which depends on time if a time-dependent external field $F(t)$ is applied to the system. Then the system evolves into the quantum state $\hat{\rho}(t) = \hat{U}_F(t,t_i)\hat{\rho}_i\hat{U}_F^{\dagger}(t,t_i)$ at time t [16,17], where the time evolution operator $\hat{U}_F(t)$ is given by

$$\hat{U}_F(t,t') = \mathrm{T}\exp\left(-\frac{i}{\hbar}\int_{t'}^{t} ds\,\hat{H}(s)\right). \tag{1}$$

In this equation, the symbol T stands for the chronological ordering of operators. We perform measurement on the system at time t_m. When we obtain some measurement outcome denoted as a, the system changes into the quantum state $\hat{\rho}_a(t_m)$ which depends on the result a [16]. After the measurement, the system evolves again with time and thus we have $\hat{\rho}_a(t) = \hat{U}_F(t,t_m)\hat{\rho}_a(t_m)\hat{U}_F^{\dagger}(t,t_m)$ at time t $(> t_m)$. Finally we perform a post-selection of the system at time t_f by making use of a generalized measurement described by positive operator-valued measure (POVM) $\hat{\pi}_f$ which satisfies $\sum_f \hat{\pi}_f = \hat{1}$ and $\hat{\pi}_f > 0$ [16,18]. In the following, we derive the average value obtained from the measurement outcomes under the condition that the post-selection $\hat{\pi}_f$ of the system is done.

2.1. Strong measurement

First we consider the case that ideal strong measurement of an observable $\hat{A}$ is performed on the system at the measurement time t_m $(t_i < t_m < t_f)$ [1,2]. It is described in terms of the projection operators $|a\rangle\langle a|$, where $|a\rangle$ is an eigenstate of $\hat{A}$ such that $\hat{A}|a\rangle = a|a\rangle$. For the sake of simplicity, we assume that all the eigenvalues are non-degenerate and thus the completeness $\sum_a |a\rangle\langle a| = \hat{1}$ and the orthogonality $\langle a|a'\rangle = \delta_{aa'}$ hold. In this case, the probability of the measurement outcome a is given by $P_s(a) = \langle a|\hat{\rho}(t_m)|a\rangle$ and the post-measurement state of the system is $\hat{\rho}_a(t_m) = |a\rangle\langle a|$ [1,2]. Then the state of the system just before the post-selection becomes $\hat{\rho}_a(t_f) = \hat{U}_F(t_f,t_m)|a\rangle\langle a|\hat{U}_F^{\dagger}(t_f,t_m)$. Since the conditional probability that the post-selection $\hat{\pi}_f$ is done for given a is $P_s(f|a) = \mathrm{Tr}[\hat{\pi}_f\hat{\rho}_a(t_f)]$, the joint probability $P_s(f,a) = P_s(f|a)P_s(a)$ of the post-selection f and the measurement outcome a is given by

$$P_s(f,a) = \mathrm{Tr}\big[\hat{\pi}_f\hat{\mathcal{U}}_F(t_f,t_m)\big\{|a\rangle\langle a|\big\{\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i\big\}|a\rangle\langle a|\big\}\big], \tag{2}$$

with

$$\hat{\mathcal{U}}_F(t,t') = \mathrm{T}\exp\left(\int_{t'}^{t} ds\,\hat{L}(s)\right), \tag{3}$$

where $\hat{L}(t) = -(i/\hbar)\hat{H}^{\times}(t)$ is the Liouvillian superoperator [17] with $\hat{X}^{\times}\hat{Y} = [\hat{X},\hat{Y}]$. In Eq. (2), Tr stands for the trace operation over the Hilbert space of the system. Using the Bayesian theorem [19,20], we obtain the posterior probability $P_s(a|f) = P_s(f,a)/\sum_a P_s(f,a)$ that the measurement outcome a is obtained under the condition that the post-selection $\hat{\pi}_f$ is done,

$$P_s(a|f) = \frac{\langle a|\hat{\mathcal{U}}_F^{\dagger}(t_f,t_m)\hat{\pi}_f|a\rangle\langle a|\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i|a\rangle}{\sum_a\langle a|\hat{\mathcal{U}}_F^{\dagger}(t_f,t_m)\hat{\pi}_f|a\rangle\langle a|\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i|a\rangle}, \tag{4}$$

where we have used $\mathrm{Tr}[\hat{X}\hat{\mathcal{U}}_F(t,t')\hat{Y}] = \mathrm{Tr}[\hat{Y}\hat{\mathcal{U}}_F^{\dagger}(t,t')\hat{X}]$. Hence we find the strong value A_s of the observable $\hat{A}$ of the pre- and post-selected system,

$$\begin{aligned} A_s^F &= \sum_a aP_s(a|f) \\ &= \frac{\sum_a a\langle a|\hat{\mathcal{U}}_F^{\dagger}(t_f,t_m)\hat{\pi}_f|a\rangle\langle a|\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i|a\rangle}{\sum_a\langle a|\hat{\mathcal{U}}_F^{\dagger}(t_f,t_m)\hat{\pi}_f|a\rangle\langle a|\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i|a\rangle}, \end{aligned} \tag{5}$$

which is equivalent to the special case of the general formula derived by Aharonov, Bergmann and Lebowitz [3,21,22].

2.2. Weak measurement

Next we suppose that weak measurement of an observable $\hat{A}$ [4–6] is performed on the system at the measurement time t_m between the pre-selection and the post-selection. The weak interaction between the system and the measuring device is usually assumed to be $\hat{H}_M = \hbar g\delta(t-t_m)\hat{A}\otimes\hat{P}$ [1], where $\hat{P}$ is a conjugate variable of a pointer observable $\hat{Q}$ of the measuring device and the commutation relation $[\hat{Q},\hat{P}] = i\hbar$ holds. In this case, the quantum state of the measuring device just after the post-selection is given by

$$\rho_M^f = \frac{\mathrm{Tr}[\hat{\pi}_f\hat{\mathcal{U}}_F(t_f,t_m)\{e^{-ig\hat{A}\otimes\hat{P}}\{\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i\}\otimes\hat{\rho}_M e^{ig\hat{A}\otimes\hat{P}}\}]}{\mathrm{Tr}_M\,\mathrm{Tr}[\hat{\pi}_f\hat{\mathcal{U}}_F(t_f,t_m)\{e^{-ig\hat{A}\otimes\hat{P}}\{\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i\}\otimes\hat{\rho}_M e^{ig\hat{A}\otimes\hat{P}}\}]}, \tag{6}$$

where $\hat{\rho}_M$ is the initial state of the measuring device at the time t_m and Tr_M is the trace operation over the Hilbert space of the measuring device. The average shift of the pointer observable caused by the interaction with the relevant system is given by $\Delta Q = \mathrm{Tr}_M[\hat{Q}(\hat{\rho}_M^f - \hat{\rho}_M)]$. In the weak coupling limit, the average pointer shift ΔQ is proportional to the real part of the weak value A_w^F of the observable $\hat{A}$ [10],

$$A_w^F = \frac{\mathrm{Tr}[\hat{\pi}_f\hat{\mathcal{U}}_F(t_f,t_m)\hat{A}\hat{\mathcal{U}}_F(t_m,t_i)\hat{\rho}_i]}{\mathrm{Tr}[\hat{\pi}_f\hat{\mathcal{U}}_F(t_f,t_i)\hat{\rho}_i]}, \tag{7}$$

the imaginary part of which can be derived from $\Delta P = \mathrm{Tr}_M[\hat{P}(\hat{\rho}_M^f - \hat{\rho}_M)]$ [10]. When the denominator on the right-hand side of Eq. (7) is sufficiently small, $\mathrm{Re}\,A_w^F$ and $\mathrm{Im}\,A_w^F$ can take large values beyond the spectral range of the observable $\hat{A}$, though the success probability of the post-selection becomes sufficiently small [4–6]. In the next section, we will investigate the linear response of the strong value A_s^F and the weak value A_w^F to an external field $F(t)$.

3. Linear response to an external field

To investigate the linear response to an external field, we assume that a weak c-number external field $F(t)$ is applied to the system during the time evolution. The interaction Hamiltonian is given by $\hat{H}_F(t) = -F(t)\hat{B}$ [11], where $\hat{B}$ is a system observable coupled to the external field. The total Hamiltonian of the system is $\hat{H}(t) = \hat{H} - F(t)\hat{B}$, where $\hat{H}$ is the Hamiltonian of the system in the absence of the external field. The corresponding Liouvillian superoperator is $\hat{L}(t) = \hat{L} + \hat{L}_F(t) = -(i/\hbar)\hat{H}^{\times} + (i/\hbar)F(t)\hat{B}^{\times}$. Then up to the first order with respect to the external field $F(t)$, we obtain