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Manipulating rogue wave triplet in optical waveguides through tapering

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ABSTRACT

Taking account of the results of the paper, published in [21] (Chabchoub and Akhmediev, 2013), containing experimental generation of rogue wave triplets in the water tank, we demonstrate a theoretical approach to coherently control the rogue wave triplet dynamics and spectral spread in a tapered index optical waveguide. The relative distance between the successive waves of the triplet, along both longitudinal and transverse axes, can be manipulated by modulating the tapering of the waveguide. This not only significantly enhances the possibility of observing these statistically rare events in the waveguide, but can also controllably amplify the intensity and spectral spread, the desired features for supercontinuum generation. The controlling of real Riccati parameter intrinsically arises from the allowed phase variation of the self-similar solutions of the nonlinear Schrödinger equation.

1. Introduction

Rogue waves are high amplitude pulses, with amplitude approximately three times higher than the average wave crest [1,2]. Firstly observed in the oceans [3], the investigation of these waves is now being pursued in other physical systems, such as nonlinear optics [4–6], Bose–Einstein condensates (BEC) [7], superfluids [8,9] and capillary waves [9,10]. Owing to their astonishing properties, they are being exhaustively studied [11–13]. Mathematically, these waves are well described by rational solutions of nonlinear Schrödinger equation (NLSE), which are localized in both space and time. NLSE being an integrable system, a hierarchy of its higher order rogue wave solutions can be obtained by using Darboux transformation [1,2]. After significant amount of theoretical and numerical studies, a thrust in the direction of their controllable experimental observation is catching pace. First ever work in this direction was done by Solli et al. [4,14]. They showed the existence of rogue waves in nonlinear fiber optics, and the concept of optical rogue wave was introduced as a counterpart of the oceanic rogue waves. These waves were found to be generated infrequently from initial smooth pulse, resulting from the power transfer seeded by small perturbation. The experimental ability to dilate the temporal duration with group velocity dispersion played a significant role in their observation. Rogue wave dynamics and its control is an area of active current research, due to its application in supercontinuum generation [5,14].

Recently a new phenomenon, known as 'splitting of higher order rogue waves into lower order rogue waves' is drawing significant attention [15–20]. Here, one observes that an nth order rogue wave can be decomposed into $n(n+1)/2$ Peregrine breathers and hence these are also called fissioned higher order rogue waves. To the lowest order, this effect can be observed in the second order rogue wave, where its splitting into three first order rogue waves takes place, well known in the literature as rogue wave triplet. This signifies their nonlinear superposition; if it would have been a linear superposition phenomenon, then for the nth order rogue wave, number of peaks would have been n. The triplet rogue wave comprises a two parameter family [15]. These real free parameters define size and orientation of the Peregrine breather. These are implicitly related to the translation of spatio-temporal axes. When these parameters take zero values, the solution gets localized at origin with the amplitude five times that of an average crest. For arbitrary nonzero values of these parameters, a triplet of rogue waves, distributed on equilateral triangle, is formed [16,17], where the distance between the peaks depends on their magnitude. First experimental observation of these rogue wave triplets took place in water tank [21]. Chabchoub and Akhmediev

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performed it specifically for two sets of these parameters, although in principle these can be observed for a whole family of two parameters [21]. They remarked that "the major difficulty of the experiment with the second order triplet solution is that it requires long propagation distances. These are longer than the actual length of the tank which is a significant limitation. To overcome this difficulty the experiment can be done in sequences". Further they write "this work may also motivate similar experiments in optics. An advantage of the fiber optical setup is that the arbitrarily large propagation distances can be realized thus avoiding the restrictions that we have in the case of a short water tank".

The fact that nonlinear superposition plays an intrinsic role in the generation and dynamics of the rogue wave triplet, raises the possibility of their manipulation through phase control. Here, we show the coherent control of rogue wave triplets in a tapered index optical waveguide, which makes essential use of their phase manipulation. Due to which not only the relative distance between the triplets can be varied controllably, but their intensity can also be manipulated through superposition effect. We find that by modulating the tapering of waveguide, intensity can be enhanced which also results in their closer appearance. This is significantly helpful in observing the rogue wave triplet, a statistically rare event, in a smaller length experimental set up. In our study, we mainly focus on sech^2 type tapered waveguide and the untapered waveguide for comparison. The present work is an extension of our previous work regarding the control of the intensities of similaritons and rogue waves in the tapered graded-index nonlinear waveguide [22,23]. In this context, various research groups have done an exhaustive study of the generalized NLSE describing the BEC, as well as nonlinear optics [24–30]. Recently Dai et al., studied the existence of self-similar rogue wave triplet in nonlinear system with linear potential and studied the effect of postponement, recurrence and annihilation of these waves, in the presence of tapering and inhomogeneous nonlinearity structure [31]. Further, for the periodically distributed systems they showed the periodic recurrence of rogue wave triplet resulting in the formation of cluster [32].

As for the homogeneous/autonomous NLSE case, distance between waves of triplet can be controlled by the free parameters [15]. Here in this work we show that by keeping the values of these parameters same but modulating the tapering of waveguide, this distance can be controlled quite effectively. The introduced parameter hence can be physically interpreted as the modulation in tapering profile of nonlinear optical waveguide. In this process, their intensities and spectral spread can also be manipulated, a much desired feature in supercontinuum generation.

The Letter is organized as follows. In Section 2, we briefly outline the governing equation for the tapered waveguide. Establishing its connection with pure NLSE, we illustrate the procedure of phase control of the rogue waves through tapering modulation. In Section 3, different types of tapering are introduced. After comparing the dynamics of rogue wave triplet in tapered and untapered waveguides in Section 4, we summarize our results and make the conclusion (Section 5).

2. Model system and governing equation

The dynamics of system under study is described by generalized NLSE

$$i\frac{\partial u}{\partial z}+\frac{1}{2k_0}\frac{\partial^2 u}{\partial x^2}+\frac{1}{2}k_0 n_1 F(z)x^2 u-\frac{i(g(z)-\alpha(z))}{2}u + k_0 n_2|u|^2 u=0, \tag{1}$$

where $u(x,z)$ is the complex envelope of the electric field, g and α account for linear gain and loss, respectively. Parameter $k_0 = 2\pi n_0/\lambda$, λ being the wavelength of the optical source generating the beam; n_1 is the linear defocussing parameter ($n_1>0$), and n_2 represents Kerr-type nonlinearity. The dimensionless profile function $F(z)$ can be negative or positive, depending on whether the graded-index medium acts as a focusing or defocussing linear lens. We introduce the normalized variables $X=x/w_0$, $Z=z/L_D$, $G=[g(z)-\alpha(z)]L_D$, and $U=(k_0|n_2|L_D)^{1/2}u$, where $L_D=k_0 w_0^2$ is the diffraction length associated with the characteristic transverse scale $w_0=(k_0^2 n_1)^{-1/4}$. Thus Eq. (1) can be rewritten in a dimensionless form [33]:

$$i\frac{\partial U}{\partial Z}+\frac{1}{2}\frac{\partial^2 U}{\partial X^2}+F(Z)\frac{X^2}{2}U-\frac{i}{2}G(Z)U+|U|^2U=0. \tag{2}$$

The self-similar optical rogue wave solutions of Eq. (2) can be obtained by transforming it into standard NLSE by using gauge and similarity transformations [33,32], together with generalized scaling of Z variable

$$U(X,Z)=\frac{1}{W(Z)}\Psi\left[\frac{X-X_c(Z)}{W(Z)},\zeta(Z)\right]e^{i\Phi(X,Z)}, \tag{3}$$

where, $W(Z)$ and $X_c(Z)$ are the dimensionless width and position of the self-similar wave center. The quadratically chirped phase is given by

$$\Phi(X,Z)=C_1(Z)\frac{X^2}{2}+C_2(Z)X+C_3(Z), \tag{4}$$

where $C_1(Z)$, $C_2(Z)$ and $C_3(Z)$ are parameters related to the phase-front curvature, the frequency shift, and the phase offset, respectively. Using Eqs. (3) and (4) in Eq. (2), we obtain $\Psi(\zeta,\chi)$ satisfying

$$i\frac{\partial \Psi}{\partial \zeta}+\frac{1}{2}\frac{\partial^2 \Psi}{\partial \chi^2}+|\Psi|^2\Psi=0, \tag{5}$$

with the effective propagation distance, similarity variable, guiding-center position, and phase are given respectively as

$$\zeta(Z)=\zeta_0+\int_0^Z\frac{dS}{W^2(S)},\qquad \chi(X,Z)=\frac{X-X_c(Z)}{W(Z)},$$

$$X_c(Z)=W(Z)\left(C_{02}\int_0^Z\frac{dS}{W^2(S)}+X_0\right),$$

and

$$\Phi(X,Z)=\frac{X^2}{2W(Z)}\frac{dW(Z)}{dZ}+\frac{C_{02}X}{W(Z)}-\frac{C_{02}^2}{2}\int_0^Z\frac{dS}{W^2(S)}, \tag{6}$$

where, $C_2(0)=C_{02}$, $X_c(0)=X_0$, with $W(0)=1$. Further, the tapering function, gain and width $W(Z)$ of self-similar wave are related as

$$d^2W(Z)/dZ^2-F(Z)W(Z)=0, \tag{7}$$

and

$$G(Z)=-\frac{1}{W(Z)}\frac{dW(Z)}{dZ}. \tag{8}$$

Hence, for any given solution of NLSE, the corresponding self-similar solutions of Eq. (2) can be obtained using the transformation (3). Among all existing solutions of NLSE, we are specifically interested in its rogue wave triplet solution:

$$\Psi(\chi,\zeta)=\left[1+\frac{G_2(\chi,\zeta)+iK_2(\chi,\zeta)}{D_2(\chi,\zeta)}\right]e^{i\zeta},$$