Physics Letters A 379 (2015) 348–360

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Stability and vibrations of doubly parallel current-carrying nanowires immersed in a longitudinal magnetic field

Keivan Kiani *

Department of Civil Engineering, K.N. Toosi University of Technology, P.O. Box 15875-4416, Tehran, Iran

A R T I C L E I N F O

Article history:
Received 15 August 2014
Received in revised form 31 October 2014
Accepted 6 November 2014
Available online 13 November 2014
Communicated by R. Wu

Keywords:
Transverse vibration
Doubly parallel nanowires
Surface elasticity theory
Electric current
Longitudinal magnetic field
Biot–Savart law

A B S T R A C T

This paper deals with dynamic interactions of two parallel nanowires carrying electric currents in the presence of a longitudinal magnetic field. Using Biot–Savart law and a surface elasticity model, the equations of motion are obtained. Accounting for both Lorentz and gravity forces, the static and the purely dynamic parts of the total displacements of the nanosystem are explicitly expressed. Two crucial modes of vibration, synchronous and asynchronous patterns, are identified and their characteristics are inclusively explained. It is shown that the nanosystem becomes dynamically unstable under certain conditions in the asynchronous mode. The minimum initial tensile force as well as the maximum values of the electric current and the magnetic field strength corresponding to the dynamic instability are derived. The roles of the crucial factors on the lowest asynchronous frequencies are also addressed and discussed.

1. Introduction

Because of their superior mechanical behavior [1,2] as well as advanced physical and chemical properties [3–5], nanowires have been regarded as a hot materials in recent years. Nanowires exist in various forms which are made of metals, organic compounds, insulators, and semiconductors. These nanostructures have been widely scrutinized for potential exploitation in optics [6–8], energy conversion and storage [9,10], bioengineering [11,12], medicine [13,14], physical sensing [15,16], chemical sensing [17,18], electronics [19–21], and micro-/nano-electromechanical systems (MEMs/NEMs) [22–25]. In the latter two applications, an ensemble of current-carrying nanowires in the presence of a magnetic field may be used for the considered jobs, however, little is known on their vibrations and dynamic instabilities. Such a fact encouraged the author to explore dynamic behaviors of magnetically affected double-nanowire-systems as a starting point for better realizing vibrations of more complex systems like as networks of current-carrying nanowires.

Due to their high slenderness, nanowires are classified as one-dimensional nanostructures. The length of a nanowire is at least 20 times greater than its lateral sizes which are at the nanometer scale. When a structure becomes so thin such that its thickness reduces to nanoscale, the surface effect becomes important and should be appropriately taken into account. The surface elasticity theory of Gurtin and Murdoch [26,27] is one of the most well-known continuum theories deals with the equations of motion of the surface layers as well as their constitutive relations. So far, such a model has been extensively applied to many physical problems of nanowires including static analysis [28–30], buckling [31–33], vibrations [34–37], and postbuckling [38].

A close scrutiny of the literature shows that there exist a large body of works on the effect of the magnetic fields on vibrations of nanostructures [39–49]. In most of these works, the nanostructures were made of a highly conducting material and no electric current was passing through them. Former works on the influence of the magnetic field on the current-carrying nanowires display that how deformation of the nanowires would lead to the exertion of the Lorentz magnetic force on them [50,51]. In this regard, Kiani [50] investigated forced vibrations of current-carrying nanowires subjected to a longitudinal magnetic field. Using Galerkin and Newmark-β approaches, the equations of motion were discretized in the spatial and time domains, respectively. Additionally, the roles of the frequency of the applied load, surface effect, nonlocality, magnetic field strength, and electric current on the vibrations of the nanostructure were explored. In a complementary work, Kiani [51] addressed dynamic instability of an individual current-carrying nanowire acted upon by a longitudinal

* Tel.: +98 21 88779473; fax: +98 21 88779476.
E-mail addresses: k_kiani@kntu.ac.ir, keivankiani@yahoo.com.

http://dx.doi.org/10.1016/j.physleta.2014.11.006
0375-9601/

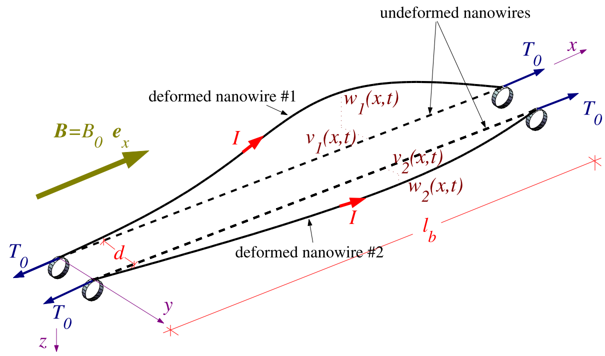

Fig. 1. Doubly current-carrying nanowires subjected to a longitudinal magnetic field.

magnetic field in the context of the surface elasticity theory. By proposing an analytical approach, the influences of the magnetic field strength, direct electric current, surface effect, and initial tensile force on the transverse displacements as well as natural frequencies of the nanostructure were explained. The above-mentioned works were restricted to dynamic analysis of a single nanowire as an electric current carrier in a magnetic field. When one confronts to a double-current-carrying nanowire-system acted upon by a longitudinal magnetic field, each nanowire is not only affected by the longitudinal magnetic field, but also by the generated magnetic field of its adjacent nanowire according to the Biot–Savart law. The resulted magnetic field plus to the longitudinally applied magnetic field yields exertion of the Lorentz forces on the nanowire. Such a fact makes some complexities in solving the problem because of the application of two magnetic fields on each nanowire in which their sources are completely different.

In this paper, an analytical solution is developed to understand the mechanism of dynamic interactions between the magnetically affected double current-carrying nanowires. The origin of the dynamic instability in such a system is also of concern. To this end, using Biot–Savart and Lorentz force laws, the interactional magnetic forces between two long elastic nanowires immersed in a longitudinal magnetic field are evaluated. By adopting an appropriate surface elasticity model, the governing equations associated with the transverse vibrations of the nanowires are derived under gravity and magnetic forces. An analytical solution is then proposed and the corresponding static and dynamic displacements of the nanosystem are obtained. The explicit expressions of the natural frequencies are obtained and their corresponding vibration modes are explained. The conditions lead to the instability of the nanostructure are inclusively displayed. The roles of the interwire distance, strength of the magnetic field, electric current, and initial tensile force on the lowest frequencies pertinent to the asynchronous pattern of vibration are comprehensively explained and discussed. It is hoped that the obtained results in this paper would give useful insights to those researchers who are interested in dynamic behaviors of magnetically affected networks of nanowires used for carrying electric current.

2. Assessment of the mutually exerted magnetic forces

Consider two long, initially straight, parallel nanowires of length l_b immersed in a longitudinal magnetic field, $\mathbf{B}_0 = B_0\mathbf{e}_x$, as illustrated in Fig. 1. These nanowires are located in a free space, separated by a distance d, and carry electric current of magnitude I in the x direction. The Cartesian coordinates system has been attached to the nanowire 1 such that the x-axis is coincident with its revolutionary axis and the z-axis is along the applied gravitational acceleration, g. The unit base vectors associated with the x, y, and z axes are denoted by $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, respectively. The displacement field vectors of the nanowires are represented by: $\mathbf{u}_i = v_i(x,t)\mathbf{e}_y + w_i(x,t)\mathbf{e}_z;\ i = 1,2$ where t is the time parameter. Because of the small deformation, the more accurate electric current vectors in these nanowires would be: $\mathbf{I}_i = I\mathbf{e}_x + I\frac{\partial v_i}{\partial x}\mathbf{e}_y + I\frac{\partial w_i}{\partial x}\mathbf{e}_z;\ i = 1,2$ where ∂ represents the partial differential symbol. In this part, we are interested in determining the magnetic force on each nanowire due to the applied magnetic fields.

Based on the Biot–Savart law, the produced magnetic field by the current-carrying nanowire 1 at a point $(x_2, d + v_2, w_2)$ of the nanowire 2 with distance r from the deformed element $d\mathbf{s}_1$ of the nanowire 1 is given by:

$$\mathbf{B}_1 = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{d\mathbf{s}_1 \times \mathbf{e}_r}{r^2}, \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}\frac{\mathrm{T\,m}}{\mathrm{A}}$ is the permeability of free space, $d\mathbf{s}_1 \approx dx(\mathbf{e}_x + \frac{\partial v_1}{\partial x}\mathbf{e}_y + \frac{\partial w_1}{\partial x}\mathbf{e}_z)$ for small deformation, $\mathbf{e}_r$ is the unit base vector pertinent to the position vector of the point with respect to the deformed element of the nanowire 1, and $r = \sqrt{(x_2 - x_1)^2 + (d + \triangle v)^2 + (\triangle w)^2}$ where $\triangle v = v_2 - v_1$ and $\triangle w = w_2 - w_1$. For long nanowires, Eq. (1) is reduced to:

$$\mathbf{B}_1 = \frac{\mu_0}{2\pi r'}\mathbf{I}_1 \times \mathbf{e}_{r'}, \tag{2}$$

where r' and $\mathbf{e}_{r'}$ in order denote the length and unit base vector of $\mathbf{r}' = (d + \triangle v)\mathbf{e}_y + \triangle w\mathbf{e}_z$. Therefore, the resulting magnetic field vector at the position of the deflected nanowire 2 is:

$$\mathbf{B}_2 = \left(B_0 + \frac{\mu_0 I}{2\pi r'}\left(\frac{\partial v_1}{\partial x}\sin\alpha - \frac{\partial w_1}{\partial x}\cos\alpha\right)\right)\mathbf{e}_x - \frac{\mu_0 I}{2\pi r'}\sin\alpha\mathbf{e}_y + \frac{\mu_0 I}{2\pi r'}\cos\alpha\mathbf{e}_z, \tag{3}$$

where $\sin\alpha = \frac{\triangle w}{r'}$ and $\cos\alpha = \frac{d+\triangle v}{r'}$. Based on the Lorentz force law, the exerted magnetic force per unit length of the nanowire 2 is evaluated by:

$$\mathbf{f}_2 = \mathbf{I}_2 \times \mathbf{B}_2, \tag{4}$$