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Entanglement and quantum phase transition of spin glass: A renormalization group approach

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ABSTRACT

Using a renormalization group approach, we study the entanglement properties of two spin glass models: the XXZ Heisenberg (with Dzyaloshinskii–Moriya interaction) and Ising transverse field spin glasses. The concurrence for both models are obtained through the Kadanoff renormalization group (RG) approach with random J_i^z and J_i respectively. The constant couplings in the RG flow are randomized through the Gaussian distribution. For $\Delta = 0$ corresponding to a non-spin glass material, a first-order transition is expected. By varying Δ from 0.05 to 0.5, the spin glass effect broadens the sharp transition resulting in a second-order-like transition. The fluctuations in the average concurrence for the spin glass case as measured by the standard deviations is also a good indicator of quantum phase transition.

1. Introduction

With just a few percent of a magnetic element randomly distributed in a non-magnetic host such as noble metals, a dilute alloy of a spin glass is formed producing many interesting experimental results. These results have initiated a whole range of new topics specifically in the areas of statistical mechanics and condensed matter physics. Unlike a classical piece of glass, a spin glass consists of magnetic moments or spins which are randomly distributed and quenched. Due to the disorderness, the spins conflict with one another giving rise to frustration effects [1–7]. These disorders and frustrations produce a complex and rugged free energy landscape. The magnetic element or impurity like manganese (Mn), iron (Fe) or europium (Eu) is introduced into a non-magnetic metal capable of dissolving the impurities. Examples of such diluted alloys are copper and manganese, $Cu_{1-x}Mn_x$ [8] or gold and iron, $Au_{1-x}Fe_x$ [9]. Alloys with properties of insulation and conduction can also be considered as spin glass and there are europium strontium sulfur $Eu_xSr_{1-x}S$ [10] which is a semiconductor and lanthanum gadolinium aluminum $La_{1-x}Gd_xAl_2$ [11] which is a metal.

Two of the central experimental signatures that are usually used to characterize whether a material is a spin glass are magnetic susceptibility and specific heat capacity. For any typical spin glass, the magnetic susceptibility usually shows a cusp at a certain freezing temperature T_f for low applied magnetic field. By varying the impurity concentration in these alloys, there is a critical temperature which corresponds to the cusp of the susceptibility. This critical temperature is termed the freezing temperature [12]. For any phase transition to occur, all thermodynamic functions will behave singularly [13,7]. Hence, the cusp in the magnetic susceptibility suggests that there may be a phase transition at a particular critical temperature. A broader maxima is produced if around 100 G of applied magnetic field is present when the susceptibility is measured [9,14,15]. In contrast to the effect of being field dependent, certain spin glasses are also found to be frequency dependent [8,16]. Even though the magnetic susceptibility of a typical spin glass does exhibit a sharp cusp in low magnetic field, other measurements like the specific heat capacity of $Au_{0.92}Fe_{0.08}$ [17] and CuMn [18] were found to have no singularity. This means that only a broad, smooth and rounded maximum is produced instead of a cusp. Moreover, the rounded maximum of the specific heat capacity does not coincide with the transition temperature for the magnetic susceptibility. Beyond the experimental studies, theories like the Edwards–Anderson (EA) model [19] which only allows the spins to interact via nearest-neighbor couplings with no long range order and Sherrington–Kirkpatrick (SK) model [20] for which every spin couples equally with every other spin are formulated in an attempt to explain mainly the cusp in the magnetic susceptibility. The EA model essentially replaces the site disorder and Ruderman–Kittel–Kasuya–Yosida (RKKY) distribution [21–23] with a random set of bonds. This set of random bonds is usually taken from a distribution like Gaussian. In order to understand

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the spin glass phase, an order parameter q has been formulated to characterize it. Despite that different and new theories have been produced to understand the physics of spin glass, other problems from the theories have since arisen. As an example, original EA equations are not simple to solve and are only soluble in the limits $T \to 0$ and $T \to T_f$. Moreover, the EA equations showed an asymmetric cusp in the magnetic susceptibility and specific heat. In disagreement, the results by Fischer [24] has showed that the theoretical specific heat is different from the experimental result except for the low temperature linear dependence when using spin $S = \frac{1}{2}$. Even though the SK model did exhibit a cusp in the magnetic susceptibility and specific heat, the entropy S becomes negative at $T = 0$ [20]. When $q = 0$, the spin glass state has lower free energy than it has for $q \neq 0$. With such instability in the SK solution, Almeida and Thouless (AT) [25] showed the stability limits of the SK solution by using the AT line to divide the unstable and stable areas in the spin glass phase diagram. The instability is essentially due to the fact that the SK model treats all the replicas indistinguishably. Fortunately, Parisi [26–30] came out with a replica symmetry breaking (RSB) scheme which removes the unphysical negative entropy. However, it was found to be at least marginally stable. Although there are some success in using these models to understand the behaviors of the spin glass, they are unable to account for all the experimental results shown. One possible reason is that these theories are classical in nature and did not consider the quantization of the spins of the impurities [16]. Nevertheless, new insights and mathematical tools developed in this field have been found to be applicable in other areas of condensed matter [6,5,31], complex optimization problems [32] and biological problems [33]. Over the recent years, $LiHo_xY_{1-x}F_4$ which can be described with a quantum Ising spin glass model has been experimentally and numerically studied [34–39]. For an x concentration of ≤ 0.25, it is believed that a spin glass phase exists. However, it is still an open question of whether a spin glass or an antiglass spin phase exists at lower concentration.

The theory of entanglement has been studied and used in both quantum information theory and condensed matter physics. In condensed matter physics, entanglement has been used to study the phase transitions of spin chain at low and finite temperature [40]. For a quantum phase transition, the change occurs at zero temperature where only the quantum fluctuations are involved and not the thermal counterpart. Numerous studies were carried out in investigating the role of entanglement in the proximity of quantum critical point for the different spin chain models [40–50]. In quantum information theory, entanglement is viewed as an important resource in applications such as quantum key distribution, quantum teleportation, quantum dense coding, entanglement swapping and others [51–58]. In particular with the application of entanglement in spin chains, density-matrix renormalization group (DMRG) approach has been utilized to understand the quantum effects for finite spin chain [59–64]. Even though such approach has been useful and accurate in describing the ground states for finite chain especially for one dimensional case, it is numerical in nature and not many works have studied using DMRG for spin glass. Other works have focused on using the Kadanoff block approach in understanding the renormalization of entanglement and phase diagram for the various spin model [65–68]. Since Kadanoff block allows one to investigate the critical behavior of the spin chain analytically, one would be curious to know if it could help us in using this approach to better understand the physics of spin glass. With this motivation, we use the Kadanoff block approach to obtain the scaled couplings from the effective Hamiltonian of the renormalized *XXZ* Heisenberg with Dzyaloshinskii–Moriya (DM) interaction and the Ising model with transverse field (ITF). With these new effective Hamiltonians containing the renormalized couplings, we investigate the entanglement of these models to finite chain with increasing renormalized group (RG) iterations. The effective couplings are then used to explore the behavior of a spin glass for finite chain.

The paper is organized as follows. We begin in Section 2 by defining the Hamiltonian for an *XXZ* Heisenberg model with DM interaction. Using this model, we defined a single RG block and the effective Hamiltonian expressed in terms of the new renormalized coupling constant J_i^z and the DM interaction D. In addition, we also look at the Ising model and obtained the new renormalized coupling constant J and the applied magnetic field B. For both models, we use the Kadanoff block approach to find the projection of each operators in the renormalized space and obtain the projected intra- and inter-block for the new effective Hamiltonian. With the use of a bipartite measure, we use the renormalized expressions to compute the entanglement (concurrence) for the block. With iteration of nth steps, we trace the RG flow and reached a steady point for finite size of spin chain. By using the rescaled renormalization equations, we explore how the concurrence changes with each iteration for the case of a spin glass where the couplings is subject to a Gaussian distribution. These results are presented and discussed in Section 3. In Section 4, we summarize our results.

2. Theoretical formulation for RG approach

2.1. *XXZ* Heisenberg model with DM interaction

In this subsection, we show how the Kadanoff approach is used in obtaining the renormalized couplings by comparing the inter- and intra-block of an *XXZ* Heisenberg spin chain. The renormalized couplings are obtained by building the projection operators on each block and projecting each block onto the lower energy subspace. The projected inter- and intra-block are mapped to an effective Hamiltonian which can then be compared to the original Hamiltonian. The *XXZ* Heisenberg model with the DM interaction in the z direction for N sites is in general given as

$$H_{XXZ} = \frac{J}{4}\left[\sum_{i=1}^{N}\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + J_i^z\sigma_i^z\sigma_{i+1}^z\right) + D\left(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x\right)\right] \quad (1)$$

where J is the constant coupling between the individual sites in the *XXZ* periodic chain and σ_i^α is the Pauli matrices ($\alpha = x, y, z$) for the ith spin [40]. The D term stands for the strength of the DM interaction along z axis and the easy-axis anisotropy is represented with J_i^z which is random. The J_i^z are quenched random variables with a probability distribution $P(J_i^z) = \frac{1}{\sqrt{2\pi}\Delta} e^{-(J_i^z-\mu)^2/2\Delta^2}$ where Δ is the standard deviation for the distribution. In general, the effective Hamiltonian H' is

$$H' = H'_B + H'_{BB} \quad (2)$$

where H_B represents the Hamiltonian for the intra-block after projection and H_{BB} represents the Hamiltonian for the inter-block after projection. We find the effective Hamiltonian by considering three qubits as a single block H_B. The coarse graining of the degrees of freedom – from 3 sites for each single block is converted into a single site which in return form another single block with other 2 sites. This process of coarse graining is shown in Fig. 1(f). In order to take into account the J_i^z coupling between the third and forth site, we need to consider the effective Hamiltonian for two single blocks. The first single intra-block I which consists of the first three sites is