Physics Letters A 378 (2014) 2790–2794

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Effects of Rashba spin–orbit coupling and a magnetic field on a polygonal quantum ring

Han-Zhao Tang [a], Li-Xue Zhai [a], Man Shen [a,b,*], Jian-Jun Liu [a,c,**]

[a] Physics Department and Hebei Advanced Thin Film Laboratory, Hebei Normal University, Shijiazhuang 050024, Hebei, People's Republic of China
[b] SKLSM, Institute of Semiconductors, Chinese Academy of Sciences, P.O. Box 912, Beijing 100083, People's Republic of China
[c] Physics Department, Shijiazhuang University, Shijiazhuang, Hebei 050035, People's Republic of China

ARTICLE INFO

Article history:
Received 13 February 2014
Received in revised form 23 July 2014
Accepted 24 July 2014
Available online 30 July 2014
Communicated by R. Wu

Keywords:
Polygonal quantum ring
Magnetic field
Rashba spin–orbit coupling
Spin transport

ABSTRACT

Using standard quantum network method, we analytically investigate the effect of Rashba spin–orbit coupling (RSOC) and a magnetic field on the spin transport properties of a polygonal quantum ring. Using Landauer–Büttiker formula, we have found that the polarization direction and phase of transmitted electrons can be controlled by both the magnetic field and RSOC. A device to generate a spin-polarized conductance in a polygon with an arbitrary number of sides is discussed. This device would permit precise control of spin and selectively provide spin filtering for either spin up or spin down simply by interchanging the source and drain.

1. Introduction

In recent years, there has been great interest in quantum computation and in applications of low dimensional semiconductor devices in which spin-dependent transport phenomena are apparent [1,2]. Datta and Das proposed a spin-field-effect transistor device for controlling the spin of an electron [3]. When an electron is transmitted from the source to the drain in this device, the spin precession and the spin-dependent phase of the electron could be controlled by the Rashba spin–orbit coupling (RSOC), which could be controlled by a perpendicular electric field [3]. In addition, both the Aharonov–Bohm (AB) [4] and Aharonov–Casher (AC) [5] effects have been studied for quantum rings both experimentally and theoretically and may also be useful for controlling the spins of electrons [6–8].

Recently, high quality semiconductor rings have been made, and have attracted significant attention because of the interesting interference phenomena that arise due to their geometric structure [9]. The spin-dependent conductance of a circular quantum ring system with RSOC have been studied by Aeberhard et al. [10]. Molnár et al. [11] have studied the magnetoconductance of a chain of circular rings including the effects of a magnetic field and the RSOC. They found that the magnetoconductance can be periodically modulated by the RSOC and the magnetic field. Besides the work on circular ring models [9–13], other geometries have also been proposed and studied, especially polygonal models [14–23]. The polygonal quantum model is the basis of the quantum network model, and the spin transport properties of a polygonal quantum ring are typical properties of quantum networks. The quantum transport properties and the electron localization phenomenon of polygons model have been discussed by Bercioux et al. [15]. Very recently, Bercioux et al. [16] have studied the localization of the electron wave function in a quantum network with both RSOC and an external magnetic field. They focused on the interplay of a magnetic field and the RSOC in the quantum network. Subsequently, the quantum transport properties of a polygonal ring with an arbitrary number of sides, subjected to RSOC, have been reported [14,22,23]. The spin transport properties of a regular polygon with RSOC have also been studied [19]. However, the magnetoconductance and the spin transport properties of a polygonal with both the RSOC and the magnetic field have not been discussed. It is now known that the transmitted electrons in a polygon should show AB oscillation phenomena which can be controlled by a magnetic flux. The magnetic flux is a function of the shape of polygonal model. In addition, time reversal symmetry can be broken by the magnetic field. Likewise, an electron in a ring with RSOC may have a spin-dependent phase that can be controlled by an adjustable gate

* Corresponding author at: Physics Department and Hebei Advanced Thin Film Laboratory, Hebei Normal University, Shijiazhuang 050024, Hebei, People's Republic of China.
** Corresponding author at: No. 288, Zhufeng Street, Shijiazhuang, Hebei 050035, People's Republic of China. Tel.: +86 311 66617006; fax: +86 311 66617006.
E-mail addresses: shenman@semi.ac.cn (M. Shen), liujj@mail.hebtu.edu.cn (J.-J. Liu).

http://dx.doi.org/10.1016/j.physleta.2014.07.039
0375-9601/

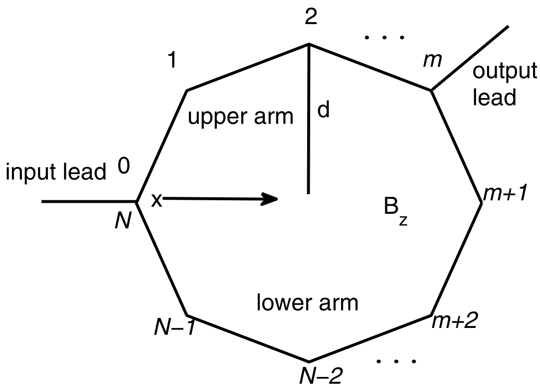

Fig. 1. Schematic diagram of the one dimensional (1D) polygonal quantum ring coupled to two leads. The RSOC and a perpendicular magnetic field B_z are applied to both the upper arm (from node 0 to node m) and to the lower arm (from node m to node N).

voltage. Therefore, the spin conductance in a polygonal ring, subjected to both RSOC and a magnetic field, will be greatly changed. The aim of in this Letter is two-fold. First, we have constructed a theoretical model to investigate analytically the spin transport properties of polygonal quantum rings with both RSOC and a magnetic field, concentrating on AB oscillations and the breaking of time reversal symmetry by the magnetic field. Second, our analysis may inspire the design of effective spin devices, spin flip and spin filtering, based on polygonal ring with both the RSOC and the magnetic field.

In this Letter, using a standard quantum network method, we analytically study the spin-dependent conductance of electrons through a polygonal ring. This Letter is composed as follows. We present the theoretical framework and derive appropriate formulas in Section 2. Section 3 presents our results and analyses. Section 4 is devoted to a summary of our work.

2. Model and formulas

A regular polygon with N segments coupled to input and output lead is shown in Fig. 1. The magnetic field and RSOC are not considered in the leads. The output lead can be located at either a symmetric or asymmetric vertex with respect to the x axis. For simplification of the analysis, we use the same approximation as in Ref. [19] and Ref. [24] for the segment of the quantum ring.

The Hamiltonian on a segment lying in an arbitrary direction $\boldsymbol{\gamma}$ and the wave functions on the input/output lead can be written in the following form [25,26]:

$$\hat{H} = \frac{(p_\gamma + qA_\gamma)^2}{2m^*} + \frac{\hbar k_{so}}{m^*}(p_\gamma + qA_\gamma)(\boldsymbol{\sigma} \times \mathbf{z}) \cdot \boldsymbol{\gamma}, \tag{1}$$

$$\psi_{in} = Ae^{ik_0 r} + Be^{-ik_0 r}, \qquad \psi_{out} = Ce^{ik_0 r}, \tag{2}$$

where A_γ is the vector potential and the meaning of other physical quantities are as in to Ref. [19]. We neglect the Zeeman splitting introduced by the magnetic field [16]. The wave function of one segment can be expressed as:

$$\begin{aligned}\Psi_{\alpha\beta}(l) = {}& \frac{e^{-if_{\alpha l}}e^{ik_{so}l(\boldsymbol{\sigma}\times\mathbf{z})\cdot\boldsymbol{\gamma}_{\alpha\beta}}}{\sin(kl_{\alpha\beta})}\big\{\sin\big[k(l_{\alpha\beta}-l)\big]\Psi_\alpha \\ &+ \sin(kl)e^{if_{\alpha\beta}}e^{-ik_{so}l_{\alpha\beta}(\boldsymbol{\sigma}\times\mathbf{z})\cdot\boldsymbol{\gamma}_{\alpha\beta}}\Psi_\beta\big\}.\end{aligned} \tag{3}$$

Because the RSOC appears in the form of the exponentials that contain Pauli matrices in Eq. (3), there are existing methods that allow us to study quantum network problems with RSOC [27,28]. The magnetic field gives rise to the phase factors

$$\begin{aligned}\exp\{-if_{\alpha\beta}\} &= \exp\left\{-i\frac{2\pi}{\phi_0}\int_\alpha^\beta \boldsymbol{A}\cdot d\boldsymbol{l}\right\} \\ &= \exp\left\{\frac{i\pi dB_z l}{\phi_0}\sin\left(\frac{N-2}{2N}\pi\right)\right\},\end{aligned} \tag{4}$$

where B_z is the magnetic field intensity, and $\phi_0 = h/e$ is the flux quantum. We define $\phi = \int_\alpha^\beta \boldsymbol{A}\cdot d\boldsymbol{l} = -\frac{1}{2}dB_z l\sin(\frac{N-2}{2N}\pi)$. The phase due to the magnetic field is described by the exponentials that contain the AB magnetic field intensity in Eq. (4), which is the key step in generalizing existing methods for studying quantum networks in the presence of a magnetic field. With the help of the Griffith boundary conditions, we can obtain the wave function of the whole quantum network [24]. To solve for the spin-dependent conductance, we separately calculate expressions for $\psi_n^{(+)}$ and $\psi_n^{(-)}$, the wave functions for electrons in the upper arm and lower arm, respectively.

$$\prod_{i=0}^{n} e^{a_i k_{so} l}\psi_n^{(+)} = c_1 e^{in(f_{\alpha\beta}+kl)} + c_2 e^{in(f_{\alpha\beta}-kl)}, \tag{5}$$

$$\prod_{i=0}^{n} e^{b_i k_{so} l}\psi_n^{(-)} = d_1 e^{in(f_{\alpha\beta}+kl)} + d_2 e^{in(f_{\alpha\beta}-kl)}. \tag{6}$$

Thus, we can obtain the transmission probability amplitude:

$$C = \frac{2i\frac{k_0}{k}\sin(kl)a}{a^{-1}a - \big[b - e^{-2if_{\alpha\beta}}\frac{\sin[(N-m-1)kl]}{\sin[(N-m)kl]}\big]\big[b - e^{2if_{\alpha\beta}}\frac{\sin[(N-m-1)kl]}{\sin[(N-m)kl]}\big]}A, \tag{7}$$

where

$$\begin{aligned}a = {}& e^{inf_{\alpha\beta}}\frac{\sin(kl)}{\sin(mkl)}e^{-(a_m+\cdots+a_1)k_{so}l} \\ &+ e^{i(N-n-2)f_{\alpha\beta}}\frac{\sin(kl)}{\sin[(N-m)kl]}e^{-(b_{N-m}+\cdots+b_1)k_{so}l},\end{aligned}$$

$$b = 2\cos(kl) - \frac{\sin[(m-1)kl]}{\sin(mkl)} - i\sin(kl)\frac{k_0}{k},$$

$$a_n = \begin{pmatrix} 0 & -e^{-i\theta_n^+} \\ e^{i\theta_n^+} & 0 \end{pmatrix}, \qquad \theta_n^+ = \frac{(N-2)\pi}{2N} - \frac{(n-1)2\pi}{N}$$

and $\quad b_n = a_n^*$.

With the help of the Landauer–Büttiker conductance formula [29,30], the conductance in the polygonal quantum ring can be obtained as:

$$G = \frac{e^2}{h}\sum_{\sigma=\uparrow,\downarrow}|T_\sigma|^2 = G_\uparrow + G_\downarrow. \tag{8}$$

$G_\uparrow$ and $G_\downarrow$ are the spin up and spin down conductances in the output lead, respectively.

3. Results and discussions

In this section, we concentrate on the physics of Eq. (7) by numerically analyzing the spin-dependent conductance. There are two ways to control the phase of electrons transmitted in the polygon. One is through the AB magnetic flux which is described by Eq. (4), and the other is through the RSOC which can be regulated by the gate voltage. In Fig. 2, we focus on the AB oscillations of the shape-dependent conductance caused by the magnetic flux in a polygonal ring. In Fig. 2(a), we plot the conductance of a polygon for different numbers of segments (N) to describe the effect of the magnetic field when the incident electrons are unpolarized.