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Emergent collective behaviors on coopetition networks

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ABSTRACT

Cooperation and competition are two typical interactional relationships for intra-networks and inter-networks. This paper investigates the modeling of coopetition networks and the collective dynamics on such networks. The coopetition networks are firstly modeled by directed signed graphs. The evolutionary relationships among individuals on the coopetition networks are described by a neighbor-based dynamics model, which is also called multi-agent system (MAS). Then, under a weak connectivity assumption that the signed network has a spanning tree, some sufficient conditions are derived for the consensus, polarization or fragmentation behaviors of the MAS with the help of the structural balance theory. At the same time, signless Laplacian matrix and signed Laplacian matrix are introduced to analyze the collective dynamics of the MAS on coopetition networks. Finally, simulation results are provided to demonstrate the emergence of diverse collective behaviors on coopetition networks.

1. Introduction

Collective behaviors of complex systems emerge from local interactions among individuals. The interactions can be cooperative or competitive. Cooperation and coordination provide benefits, such as forage, aggregation, flocking, formation control, to the individual agents, which have been found widely in natural processes [1,2], biological swarming [3–5], human activities [6–9], and engineering applications [10–12]. However, competition is another inherent relationship between individuals. The British scientists' Charles Darwin's "natural selection" and Herbert Spencer's "survival of the fittest" emphasized the fact that competition has been observed between individuals, populations and species. For example, the global dynamical behavior of a class of nonlinear Volterra–Lotka system was analyzed for competing species and competitive cellular neurons [13,14]. The antagonistic relationship is also common in social systems, see [15,16]. In fact, competition and cooperation normally coexist in natural and engineering systems.

In the interactive living systems under consideration, individuals and organizations frequently need access to the others' information, meanwhile, attempting to pursue private gain. A typical example is the animals' group living, through which individuals cooperate to look for food or to watch for predators, but compete for limited resources [17]. Generally, competitive and cooperative interactions are frequent in large groups. Brandenburger and Nalebuff coined the term "coopetition" to describe the coexistence of competition and cooperation in the strategic management field [18]. A coopetitive game model was proposed in [19–21] by virtue of game theory for agents, which cooperate and compete simultaneously for different subjects in the same economic environment. Although some coopetition principles have been described in game theory, this paper focuses on the emergence of different collective behaviors of a group of interconnected agents on a coopetition network from the viewpoint of system dynamics theory.

For cooperative networks, a typical collective behavior is characterized by the emergence of a global consensus, in which all agents reach the same state in the long run. Consensus occurs in many biological, sociological, and physical systems, and it has been investigated formally since 1960s (see [22] and references therein). Some pioneering physical models were proposed to investigate various consensus behaviors, for example, fireflies' flashing [23], opinion agreement [24], clapping synchronization [25], phase transition of self-driven particles [26], oscillator synchronization [27]. Very recently, theoretical explanations have been provided for consensus behaviors under the framework of multi-agent systems and complex networks [28,29]. In the consensus literature, all agents have to interact cooperatively to achieve the same state on a network, which is usually modeled as a graph having agents as nodes and their pairwise collaboration as positive edges. It has been known that global consensus can be reached if and only if the directed graphs associated with multi-agent systems have a spanning tree [11,30].

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For coopetition networks, another type of consensus phenomenon, i.e., bipartite consensus or anti-synchronization, in which all agents reach a final state with identical magnitude but opposite sign, has been observed for a long time. For example, a polarization phenomenon often happens in a two-coalition community such that opposite opinions are held by two fractions [31–33]. As another well-known example in physics, an anti-phase synchronization phenomenon between two pendulum clocks was firstly observed by Huygens in the seventeenth century and reinvestigated in [34]. Anti-synchronization phenomena have also been observed in salt-water oscillators experiments and chaotic systems [35]. In order to investigate bipartite consensus, the interaction networks are generally modeled by signed graphs with positive/negative edges and the collective dynamics is modeled by multi-agent systems (MASs). The opinion formation on coopetition networks was analyzed by using the notion of structural balance in [33]. Structural balance is an important property in social network theory, which partitions signed graphs into two subgraphs such that each subgraph contains only positive edges while all edges joining different subgraphs are negative [15,36,37].

A classical example, which motivates us greatly to start this work, is the healthcare coopetition networks [38]. In such networks, there are two core hospitals and each of them has its own subnetwork. The hospitals in each subnetwork are cooperative to share the healthcare resource while the hospitals belonging to different subnetworks compete for healthcare market. Then an open problem arises: how can we describe the coopetition networks and analyze collective behaviors emerging from the coopetition interactions? In order to answer these questions, a general coopetition network is considered in this paper. Considering that agents in the coopetition network generally play different roles and some interactions may be unidirectional, the interactions between agents should be directed. Therefore, it is important to introduce some notations from directed signed graph theory to model coopetition networks [15]. As a significant characteristics of a graph, the connectedness involves the interaction relationships among agents and is critical to determine which collective pattern will emerge. When the interactions in coopetition networks are time-invariant, the weakest connectedness is that the networks have a spanning tree, which is a fundamental assumption in this paper. Furthermore, an appropriate model has to be built for the collective dynamics on coopetition networks. Since the interaction range of each agent is generally limited, a proper choice of the dynamics modeling is the neighbor-based method, as used in [26].

Our purpose in this paper is achieved according to the following three aspects: (i) The interaction network of a group of autonomous agents with cooperative and competitive relationships is described by a directed signed graph, which is a first step to investigate the collective dynamics on coopetition networks. Some algebraic properties of directed signed graphs are described to contribute the subsequent mathematical analysis of the collective behaviors. (ii) A neighbor-based collective dynamics on the coopetition network is established. Since the stability of the collective dynamics depends not only on the connectedness, but also on the structural balance property of the coopetition network, two cases, heterogeneous coopetition networks and homogeneous coopetition networks, are considered in the mathematical analysis and computer simulations. (iii) The conditions of three particular collective behaviors, namely, consensus, polarization or bipartite consensus and fragmentation, for the proposed multi-agent system are explored. Some theoretical results are obtained with mathematical proofs and demonstrated by a series of computer simulations with experimental explanations.

The remainder of this paper is organized as follows. In Section 2, coopetition networks are modeled and the collective dynamics on the networks are built. In Section 3, the convergence of the collective dynamics on homogeneous networks and heterogeneous networks is analyzed comprehensively. Some simulations are provided to demonstrate the diverse behaviors on different signed networks. Finally the paper is concluded in Section 4.

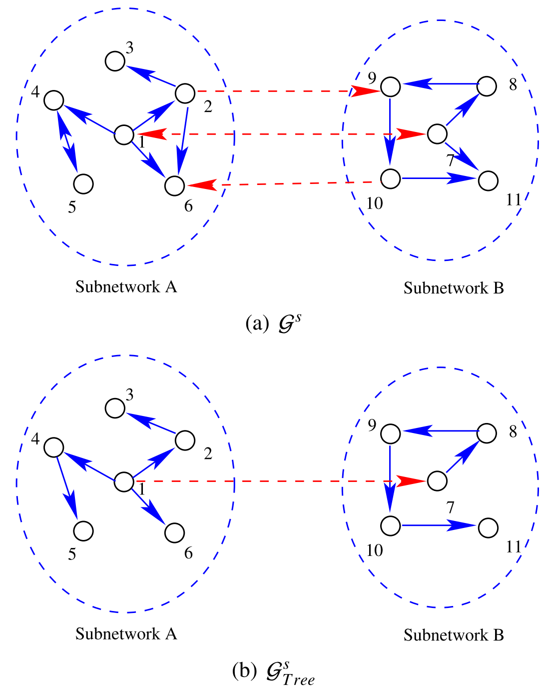

Fig. 1. Coopetition networks. In (a) and (b), the two networks $\mathcal{G}^s$ and $\mathcal{G}^s_{Tree}$ are signed graphs, which have positive and negative edges. The positive and negative edges are denoted by blue solid and red dash lines, respectively. The graph $\mathcal{G}^s_{Tree}$ is a spanning tree of $\mathcal{G}^s$ and the node 1 is a root node. The network $\mathcal{G}^s$ has two subnetworks A and B. The edges are positive within each subnetwork and are negative between the two subnetworks. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2. Problem formulation

2.1. Coopetition network modeling

When we regard an agent as a node and the interactions between two agents as directed edges, it is helpful to use directed signed graphs to describe coopetition networks. The positive and negative edges in directed signed graphs represent, respectively, the cooperative and competitive interactions in coopetition networks. Herein, we give an intuitive illustration of coopetition networks in Fig. 1, which is used to describe a healthcare coopetition network in [38].

Formally, *a directed signed graph* is a directed graph $\mathcal{G}^s = \{\mathcal{V}, \mathcal{E}, A\}$, where $\mathcal{V} = \{1, \cdots, n\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and A is an adjacency matrix describing the edge information of a positive or negative sign. The nonzero element a_{ij} of A is attached to the edge $(j, i) \in \mathcal{E}$, which is directed from node j to node i. In this way, the node j is called the parent node and i is the child node, which means that the information flow starts from agent j and ends at agent i. The edge set $\mathcal{E} = \mathcal{E}^+ \cup \mathcal{E}^-$, where $\mathcal{E}^+ = \{(j, i) \mid a_{ij} > 0\}$ and $\mathcal{E}^- = \{(j, i) \mid a_{ij} < 0\}$ are the sets of positive and negative edges, respectively. If all edges are positive and $\mathcal{E}^- = \varnothing$, the graph is simply a directed unsigned graph or digraph $\mathcal{G}$. A directed path is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \cdots, (i_{l-1}, i_l)$ with distinct nodes with length $l - 1$. A semipath is defined as a sequence of nodes $i_1, \cdots, i_l$ such that either $(i_\kappa, i_{\kappa-1})$ or $(i_{\kappa-1}, i_\kappa)$ belongs to the set $\mathcal{E}$. A directed *(semi)cycle* is a directed (semi)path beginning and ending with the same nodes. A directed graph $\mathcal{G}^s$ is said to be strongly (weakly)