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Probing spin entanglement by gate-voltage-controlled interference of current correlation in quantum spin Hall insulators

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ABSTRACT

We propose an entanglement detector composed of two quantum spin Hall insulators and a side gate deposited on one of the edge channels. For an *ac* gate voltage, the differential noise contributed from the entangled electron pairs exhibits the nontrivial step structures, from which the spin entanglement concurrence can be easily obtained. The possible spin dephasing effects in the quantum spin Hall insulators are also included.

1. Introduction

It is well known that the entanglement reflects a kind of nonlocal correlation [1–4] and plays an important role in quantum information and computation science [5]. Recently, the creation and detection of electronic entanglement in solid state systems have attracted much interest, for the large-scale implementation of quantum information and computation schemes [6]. The crossed Andreev reflection in mesoscopic s-wave superconductor systems has already been confirmed [7–9], which is regarded as an effective proposal for the generation of spin entangled electrons in solids [10,11].

Although the idea to utilize entanglement in solids is elegant, a direct experimental evidence is still challenging. Several proposals for spin entanglement detection have been put forward, including the Bell inequality tests [12–14] and the measurement of the shot noise in a beam splitter setup [15]. The former is based on the local hidden variable theories [3,4] and the latter utilizes the relation between the spin entanglement and the antisymmetry of the electron wave functions [15]. Most recently, we have suggested another detection scheme by use of the quantum eraser effect [16] and the complementarity principle [17], where the spin entanglement concurrence is measured by the Aharonov–Bohm oscillation of the current correlation [18].

On the other side, the two-dimensional quantum spin Hall insulator (QSHI) has received much attention recently, as a topological matter [19,20]. There are fully gapped bulk states and gapless helical edge states in QSHIs protected by the band topology [19,20]. The helical electrons have their spins and moving directions bounded together, which provides an opportunity for the all-electrical control of spins [14].

In this paper, we propose a spin entanglement detector constructed by two QSHIs and a side gate on one of the edge channels. When entangled electrons are injected separately into different edge channels, the interference pattern of their current correlation contains the information of the entanglement and is controlled by the side gate in an all-electrical manner. More remarkably, under an *ac* gate voltage, the differential noise exhibits a notable step structure for an easy observation and the spin entanglement concurrence can be drawn from the heights of those steps.

2. Model calculation

The proposed setup is shown in Fig. 1, where the entangler in the middle region is weakly coupled to two QSHIs. When a bias voltage is applied between the entangler and the terminals, the entangled electrons will tunnel into the helical edge channels of QSHIs. We will investigate the current correlation between the left and right terminals to reveal the entanglement information. The entangled electrons may be injected into the same QSHI or separately into different QSHIs. However, the current correlation contributed from the former process is negligibly small in the

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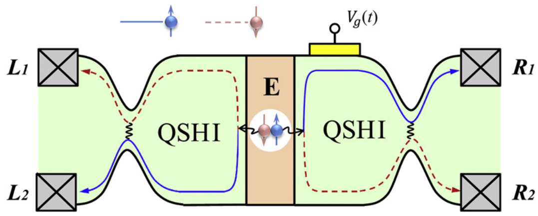

Fig. 1. (Colour on-line.) Illustration of the proposed setup. The entangled electrons are travelled separately from the entangler (labelled by E) into the helical edge channels of QSHIs. The spin-up (-down) channels are sketched as blue solid (red dashed) lines. The narrow regions in both sides represent the QPCs, which bring coupling between the upper and lower edge channels and serve as the beam splitters. A side gate is deposited on the upper channel on the right side. The currents and current correlations are measured in four terminals labelled by $L_{1,2}$ and $R_{1,2}$, respectively.

weak coupling limit [14,21] so that only the latter process will be considered below. In each QSHI, there is a quantum point contact (QPC) depicted as the narrow region in Fig. 1, where the opposite edge channels are coupled together leading to the scattering between edges. It has been demonstrated that such a QPC can serve as an ideal beam splitter without any back scattering [22]. A side gate is deposited on the upper edge of the right QSHI, which modulates the phase carried by the electron in that path [23].

The entangler can be realized by using superconductors [7–11] or quantum dots [24,25]. Here we adopt the former as an example, however, the proposed scheme is general and can be adapted to other kinds of entanglers. The superconductor is grounded and a bias voltage $-eV$ (less than the superconducting gap) is applied to all the terminals, so that the Cooper pairs are split into the helical edges in the QSHIs [26,27]. In the weak coupling limit, the tunnelling coefficient is assumed as $\chi \ll 1$. The entangled electron state is given by [14,21]

$$|\Psi\rangle = \chi \int_{-eV}^{eV} d\varepsilon \big[\sqrt{\kappa} a^\dagger_{L\uparrow}(\varepsilon) a^\dagger_{R\downarrow}(-\varepsilon) \mp \sqrt{1-\kappa} a^\dagger_{L\downarrow}(\varepsilon) a^\dagger_{R\uparrow}(-\varepsilon)\big]|0\rangle, \tag{1}$$

where $|0\rangle$ represents the filled Fermi sea with a Fermi energy $-eV$, and $a^\dagger_{i,\sigma}(\varepsilon)$ creates an electron of energy ε and spin σ moving in side i, which satisfies the anticommutation relation $\{a_{i,\sigma}(\varepsilon), a^\dagger_{j,\sigma'}(\varepsilon')\} = \delta_{i,j}\delta_{\sigma,\sigma'}\delta(\varepsilon - \varepsilon')$. The helicity of the edge states indicates the spin–edge correspondence. For example, $a^\dagger_{L\uparrow}$ describes the creation of a spin-up electron moving left along the lower edge, as shown in Fig. 1. By changing the parameter $\kappa \in [0, 1]$, the state in Eq. (1) can vary from the mostly entangled states ($\kappa = 1/2$) to the product states ($\kappa = 0, 1$). The measurement of the entanglement for a pure state can be described by the concurrence $C = 2\sqrt{\kappa(1-\kappa)}$ [28,29]. When $C = 1$, the signs "$\mp$" in Eq. (1) correspond to the singlet and triplet entangled states, respectively.

After the transmission into the QSHIs, the electrons get scattered at the QPCs and then finally reach the terminals. The QPC is made of a narrow QSHI with coupling between two edges and the controllable spin–orbit coupling so that it can be regarded as an ideal beam splitter, where the electrons may change their edges during the forward scattering while the back scattering is completely ruled out [22]. Thus, the operators for electrons in terminals L_1 and R_1 can be written as

$$a_{L_1} = t_{2L} a_{L\uparrow} + t_{1L} a_{L\downarrow},$$
$$a_{R_1} = t_{1R} a_{R\uparrow} + t_{2R} a_{R\downarrow}, \tag{2}$$

where the coefficients $t_{1,2i}$ describe the amplitudes for the same-edge and the cross-edge transmissions in side i, respectively.

In order to probe the entanglement, we calculate the zero frequency noise power between terminals L_1 and R_1, which reads

$$S = 2 \int_{-\infty}^{\infty} dt \big[\langle \hat I_{L_1}(t) \hat I_{R_1}(0)\rangle - \langle \hat I_{L_1}(t)\rangle\langle \hat I_{R_1}(0)\rangle\big], \tag{3}$$

where the current operator is defined as $\hat I_{i_1}(t) = (e/h) \times \iint dE dE' \times e^{i(E'-E)t/\hbar} a^\dagger_{i_1}(E') a_{i_1}(E)$. The average in Eq. (3) is taken under the state Eq. (1), whose magnitude is of the order of χ. Therefore, the leading term $\langle \hat I_{L_1}(t) \hat I_{R_1}(0)\rangle$ is of the order of χ^2, while the term $\langle \hat I_{L_1}(t)\rangle\langle \hat I_{R_1}(0)\rangle$ is of the order of χ^4 and can be neglected. By utilizing Eq. (2), we have

$$\langle \hat I_{L_1}(t) \hat I_{R_1}(0)\rangle = \left(\frac{e\chi}{h}\right)^2 \int_{-eV}^{eV} d\varepsilon \int_{-eV}^{eV} d\varepsilon' e^{i(\varepsilon-\varepsilon')t/\hbar} \times \big[\kappa T_{2L} T_{2R} + (1-\kappa) T_{1L} T_{1R} \mp C\sqrt{T_{2L} T_{2R} T_{1L} T_{1R}} \cos\varphi\big], \tag{4}$$

where the transmission probabilities are $T_{1,2i} = |t_{1,2i}|^2$ and the total phase for electrons accumulated in a loop trajectory is $\varphi = \text{Arg}(t^*_{2L} t_{1L} t^*_{2R} t_{1R})$.

The cosine term in Eq. (4) represents the interference effect. The right-moving electron can reach terminal R_1 by two paths. One is going in the upper edge first and then arriving in the terminal through the same-edge transmission at the QPC. The other is going in the lower edge first and then through the cross-edge transmission at the QPC. Due to the entanglement shown in Eq. (1) and the helicity of the edge states, the which-path information of the right-moving electron is registered by the spin of the left-moving one. As a result, the current in terminal R_1 would not show any interference behaviour [17]. However, the current correlation between terminals R_1 and L_1 can still show interference. This is because that the left QPC can be regarded as a quantum eraser [18,16]. After scattering at the left QPC, the left-moving electrons arrive in terminal L_1 with mixed spins so that the which-path information is erased. The strength of the interference pattern of the current correlation reveals the information of the entanglement, as shown by the cosine term in Eq. (4), which is proportional to the concurrence.

Since there is a side gate on the upper channel in the right QSHI, phase φ can be split into two parts $\varphi = \varphi_g(t) + \varphi_0$, with $\varphi_g(t)$ and φ_0 being the gate-dependent and the constant phase, respectively. Due to the helicity of the edge states, the gate voltage will not lead to any back scattering. For a slow-varying gate voltage, the period of the *ac* gate is much longer than the travelling time of the electron in the gating region and the gate-dependent phase can be well approximated by $\varphi_g(t) = eV_g(t)d/\hbar v$, where V_g is the gate voltage, d is the length of the gating region, and v is the Fermi velocity of the edge states. The constant phase is a parameter of the circuit and can be adjusted beforehand, e.g. by a *dc* side gate. Here, we adopt $\varphi_0 = 0$ for simplicity. The *ac* gate is assumed as a harmonic term $V_g(t) = V^a_g \sin\Omega t$ with the amplitude V^a_g and the frequency Ω. Phase φ can be finally expressed by $\varphi = \varphi_a \sin\Omega t$ with $\varphi_a = eV^a_g d/\hbar v$.

Phase φ is a function of period $2\pi/\Omega$ so that one can obtain the Fourier expansion

$$e^{i\varphi} = \sum_{n=-\infty}^{+\infty} J_n(\varphi_a) e^{in\Omega t}, \tag{5}$$

where $J_n(\varphi_a)$ is the Bessel function of the first kind. Inserting Eq. (5) into Eqs. (3) and (4), one obtains an elegant expression for the dimensionless differential noise (DN)